Extracting a signal about the output gap from noisy growth, unemployment and inflation

This technical annex goes through the details of the exercise described in the main body of the speech. The exercise built on that shown in Broadbent (2014). That speech noted the result from Svensson and Woodford (2003) that optimal policy in the face of incomplete information about economically relevant variables, like the output gap, is invariant to the degree of accuracy. However, the "signal extraction" formula that the policymaker follows to infer those variables is sensitive to the variance of the shocks. We extend that model to include an inflation observable, and to include shocks to the natural rate of unemployment and price markup shocks.

In our small model for the economy, the output gap, x, is autocorrelated and is subject to persistent demand shocks, d. Unemployment depends on the distributed lag of x and a white-noise disturbance u*. Output growth is a combination of changes in the output gap and a supply shock, assumed to be white noise in the growth rate, Δs, and so a random walk in the level. Inflation depends on its own lag, as well as the lagged output gap, and a white noise disturbance, ε.

$$x_t = \alpha x_{t-1} + d_t$$

$$u_t = -\gamma_1 x_{t-1} - \gamma_2 x_{t-2} + u_t^*$$

$$\Delta y_t = \Delta x_t + \Delta s_t$$

$$\pi_t = \lambda \pi_{t-1} + \kappa x_{t-1} + \varepsilon_t$$

The standard deviations of the four shocks are σ_d, σ_u, σ_s and σ_ε. We observe Δy, u and π. Our task is to find the best possible estimate of x.

The parameter values are chosen to deliver variances of the observable variables that broadly match that in the annual UK data: $\alpha = 0.25$, $\gamma_1 = 0.25$, $\gamma_2 = 0.55$, $\lambda = 0.4$, $\kappa_1 = 0.1$, $\kappa_2 = 0.25$, $\sigma_d = 0.23$, $\sigma_u = 0.15$, $\sigma_s = 0.17$ and $\sigma_\varepsilon = 0.23$. We solve the problem numerically. This gives rise to an updating equation for the output gap:

$$\hat{x}_t = \hat{x}_{t-1} + \kappa_y(\Delta y_t - \Delta \hat{y}_t) + \kappa_u(u_t - \hat{u}_t) + \kappa_\pi(\pi_t - \hat{\pi}_t)$$

Where $\Delta \hat{y}_t$, $\hat{u}_t$ and $\hat{\pi}_t$ are prior expectations of growth, unemployment and inflation respectively. The Kalman gain parameters (κ_y, κ_u and κ_π) can then be scaled by the expected variance of the observables to construct a variance decomposition. This can be interpreted as the contribution of innovations in each observable (and the covariances between them) to the estimate of the output gap.

The first panel in Chart [8] plots the contribution of activity and unemployment data to the estimated variance of the output gap, where along the x-axis we increase the standard deviation of the supply shock relative to the others. The second panel charts the contributions of unemployment and inflation to the estimated variance of the output gap, where along the x-axis we increase the standard deviation of the shock to u*.