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Contagion Accounting*

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Abstract

We provide a simple and tractable accounting-based stress-testing framework to assess loss dynamics in the banking sector, in a context of leverage targeting. Contagion can occur through direct interbank exposures, and indirect exposures due to overlapping portfolios with the associated price dynamics via fire sales. We apply the framework to three granular proprietary ECB datasets, including an interbank network of 26 large euro area banks as well as their overlapping portfolios of loans, derivatives and securities. A 5 percent shock to the price of assets held in the trading book leads to an initial loss of 30 percent of system equity and an additional loss of 1.3 percent due to fire sales spillovers. Direct interbank contagion is negligible in our analysis. Our findings underscore the importance of accurately estimating the price effects of fire sales.

Keywords: *Interbank networks, contagion, overlapping portfolios, fire sales, stress-testing.*
JEL: *C63,G01,G18,G21.*

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1 Introduction

Banking sector interconnectedness and contagion materialize in more than one way. The most obvious channel linking the health of various banks is the direct exposures between them. However, as highlighted by developments during the Great Financial Crisis and the Covid-19 crisis, the fate of seemingly disconnected and geographically dispersed institutions can still be bound together due to the importance of indirect contagion (Clerc et al. (2016)). Most important within the universe of such interconnections are exposures to the same assets (i.e. "overlapping portfolios") and to externalities driven by fire sales (Shleifer and Vishny (2011)). These channels are all part of the transmission of distress in the banking system. The importance of each contagion channel remains a contested issue and is ultimately an empirical question.

We provide a simple and tractable accounting-based stress-testing framework to assess loss dynamics in a banking sector connected via direct interbank market exposures and overlapping portfolios. The model allows for a straightforward incorporation of fire sale price dynamics in the spirit of Greenwood et al. (2015) and Duarte and Eisenbach (2020). We develop measures that allow for the computation of losses stemming from network exposures, as well as additional losses generated by fire sale dynamics. In this way, we can account for the contribution to system losses of each contagion channel. We apply our framework to granular microfinancial euro area data, and document that interbank exposures account for only a minor share of the overall losses due to contagion. Aside from the direct effect of shocks, the bulk of distress contagion is driven by the common exposures of banks that make them vulnerable to fire sales and synchronous price dislocations. The calibration of the price impact of fire sales is a critical input to determine the extent of losses.[1] Stress-testing would greatly benefit from more efforts to improve such calibrations.

The starting point is a basic framework focusing on direct exposures between banks.[2] The core of the framework builds on an accounting representation of the balance sheet of the banking system, which analytically bears a close resemblance to classic input-output analysis (Leontief, 1936).[3] We consider a banking system comprising assets and liabilities both internal and external to the interbank network, as well as deposits and equity on the liability side. We decompose losses into the initial loss due to a shock to the bank's balance sheet, the first-round effect due to losses suffered by direct counterparties, and the higher-round amplification effects. Some studies suggest, however,

[1] Our baseline calibration of price impact is based on Duarte and Eisenbach (2020), using haircuts from the net stable funding ratio (NSFR) standard from the Basel Committee on Banking Supervision.

[2] Allen and Gale (2000) provide an early theoretical contribution to the interbank network literature modeling direct interbank links. Elsinger et al. (2006) provide an early application of network theory to asses contagion in interbank systems. Drehmann and Tarashev (2013) use Shapley values to gauge the contribution of interconnected banks to systemic risk.

[3] Representing the structure of the banking system in an input-output framework was first suggested by Aldasoro and Angeloni (2015). In their paper, banks are connected via direct interbank exposures only and ranked according to different measures of systemic importance in a static fashion. In the present paper, we take a more dynamic approach based on the input-output framework by studying the propagation of actual shocks.

that direct interbank exposures do not generate substantial losses in the banking system, unless they are coupled with additional contagion channels such as fire sales and overlapping portfolios.[4]

We then add a second form of interconnectedness by including overlapping portfolios. Under marking-to-market, banks investing in the same asset would be simultaneously impacted by price movements in that asset. Conditional on price movements, this introduces another, indirect, contagion channel. Within our framework, we can decompose the contribution of each channel to loss dynamics. Importantly, the framework can be tailored to data availability. In the extreme, it is flexible enough to drill down to securities-level data and account for contagion from the bottom up.

Finally, to make overlapping portfolios bite as a contagion channel, we add price dynamics due to fire sales. We do so by combining our approach with a variation of the matrix-based fire sales model of Greenwood et al. (2015) and Duarte and Eisenbach (2020). The main assumption in that model is that, following a shock, banks sell assets to regain target leverage (Adrian and Shin (2014)). They sell assets for shocks of any size and from their entire asset holdings.[5] We also assume leverage targeting but we posit a different selling behavior. To make the stress testing more realistic, distressed banks sell assets only from the trading book.[6] It indeed seems unrealistic that distressed banks would also sell assets from the banking book, since these are labeled as to be held to maturity for regulatory purposes. To the best of our knowledge, this is the first paper that takes into account the distinction between the trading book and the banking book in studying fire sales dynamics, in addition to providing a framework that allows for accounting for different channels of contagion and that can be mapped directly to real data.

We complement the vulnerability measures developed by Greenwood et al. (2015). They define aggregate vulnerability as the percentage of aggregate bank equity that would be wiped out if banks fire sell in response to a shock to asset returns. The measure only takes into account the impact of the change in prices, but not the balance sheet impact of the asset sales. We therefore build analogous measures to quantify the balance sheet impact of fire sales.

The framework is illustrated by using three confidential European Central Bank (ECB) datasets for the 26 largest euro area banking groups, using observations for the first quarter of 2016. The balance sheet data are extracted from ECB supervisory statistics. These are also used to construct overlapping portfolios of loans and derivative positions as well as to build the network of bilateral exposures. In order to construct the overlapping portfolios of securities, we make use of two micro-financial datasets, namely the European System of Central Banks' Securities Holdings Statistics by

[4]See Cifuentes et al. (2005); Nier et al. (2007); Gai and Kapadia (2010); Caccioli et al. (2015); Glasserman and Young (2015a); Aldasoro and Faia (2016), as well as the surveys by Glasserman and Young (2015b) and Hüser (2015). With the exception of Caccioli et al. (2015), who have data on direct – but not indirect – exposures, these papers all use theoretical models. Furthermore, most studies also assume only one common asset.

[5]In a related paper, Cont and Schaanning (2017) develop a threshold model for fire sales where banks start selling assets once their leverage requirement reaches a specific threshold.

[6]For regulatory purposes, banks need to assign assets they hold to either the trading book or the banking book. The trading book is an accounting term that refers to assets held by a bank that are regularly traded.

Group and the ECB Centralised Securities Database. We distinguish between assets held in the trading and banking books and document that tradable assets make up a third of the total assets of banks in the sample. The five largest asset categories in the trading book are derivatives, long-term government bonds, loans to other financial corporations, loans to banks and long-term bonds issued by other financial corporations. Tradable assets holdings are higher for more leveraged banks.

A 5 percent shock to the price of assets held in the trading book leads to an initial loss of 30 percent of system equity and an additional loss of 1.3 percent due to fire sales spillovers.[7] Deleveraging in the interbank market on its own does not lead to large balance sheet adjustments, which confirms empirically the result of the simulation studies referenced above. Only in the presence of fire sales dynamics is there a significant balance sheet change due to deleveraging: in response to a 5 percent price decline in tradable assets – which wipes out 30 percent of system equity – banks will adjust their balance sheets in the order of 5.9 times system equity in order to restore initial leverage ratios.[8] In the presence of indirect contagion due to fire sales, there can be significant deleveraging in response to shocks.[9]

The paper is structured as follows. Section 2 presents the structure of our stress-testing framework, which is built sequentially from direct exposures up to the inclusion of price dynamics. Section 3 explains the impact and vulnerability measures for banks. Section 4 presents the data as well as descriptive statistics of the banks' balance sheets, of the network layers and of the banks' portfolios. Section 5 present results on the empirical application of the framework to the data presented in Section 4. Concluding remarks are presented in 6.

2 Accounting for contagion

2.1 Basic framework

We consider a banking system composed of n banks. The aggregated balance sheet in matrix notation[10] can be expressed as follows:

[7] Our empirical results show that distressed banks manage to restore their leverage ratios to their initial levels using their trading book only, which supports the assumption that they would only use sales from their trading book to restore their capital ratio.

[8] Note that this result does not imply a loss of 5.9 times system equity. The result is stated in terms of system equity to provide intuition on the magnitude of the effect. Expressed in terms of total assets, this is equivalent to a decline of the banking sector's assets of 28 percent. This result excludes the balance sheet adjustment due to the direct impact of the initial shock.

[9] The shock is of a size such that it leads *only* to bank distress as opposed to outright default. We focus on bank distress without reaching default in order to keep the framework linear and hence tractable and intuitive.

[10] Unless otherwise specified, we use standard notation from matrix algebra. By capital bold fonts (e.g. $\mathbf{X}$) we denote an $n \times n$ matrix with generic elements x_{ij}, whereas lower case bold fonts (e.g. $\mathbf{x}$) represent $n \times 1$ column vectors with generic element x_i. The transpose of a matrix or vector is indicated with a prime (as in $\mathbf{X}'$ or $\mathbf{x}'$). The vector $\mathbf{x}_j$ denotes the j^{th} column of matrix $\mathbf{X}$, whereas $\mathbf{x}'_i$ stands for the i^{th} row of matrix $\mathbf{X}$. The identity matrix is indicated by $\mathbf{I}$, the unit (column) vector is indicated by $\mathbf{i}$, and $\mathbf{i}_j$ stands for the j^{th} column of $\mathbf{I}$. $\mathbf{0}$ is a column vector of zeros. Finally, a lower case bold letter with a "hat" on it (e.g. $\hat{\mathbf{x}}$) denotes an $n \times n$ diagonal matrix with

$$\mathbf{e} + \mathbf{d} + \mathbf{X}'\mathbf{i} = \mathbf{X}\mathbf{i} + \mathbf{l}, \tag{1}$$

where $\mathbf{e}$ and $\mathbf{d}$ are column vectors denoting respectively, equity and deposits. $\mathbf{l}$ is a column vector of total non-interbank, i.e. "external", assets (composed of cash, loans, net securities holdings and lending to (reserves at) the central bank). $\mathbf{X}$ is the matrix of interbank gross bilateral positions, where an element x_{ij} represents lending from bank i to bank j and where by construction $x_{jj} = 0$ for all j. $\mathbf{i}$ is a unit (i.e. summation) vector of appropriate size. All magnitudes are expressed in monetary terms.

Let $\mathbf{q}$ be a vector with total bank assets/liabilities and $\hat{\mathbf{q}}$ be a corresponding diagonal matrix, such that $\hat{\mathbf{q}}\mathbf{i} = \mathbf{q}$. Then, as noted in Aldasoro and Angeloni (2015), the right hand side of Equation 1 can be written in the following form:

$$\mathbf{q} = \mathbf{X}\mathbf{i} + \mathbf{l} = \mathbf{X}\hat{\mathbf{q}}^{-1}\hat{\mathbf{q}}\mathbf{i} + \mathbf{l} = \mathbf{A}\mathbf{q} + \mathbf{l}, \tag{2}$$

where $\mathbf{A} = \mathbf{X}\hat{\mathbf{q}}^{-1}$ is the matrix of interbank positions in which each column is divided by the total assets of the borrowing bank. Hence, the columns of $\mathbf{A}$ are fractions of unity and express, for each bank, the share of funding from other banks as a ratio to total funding. Equation 2 is similar in form and interpretation to the so-called *input-output framework*, where total output of the industries in the economy (here total assets of the banking system) is equal to interindustry sales (interbank lending) and final demand (here non-interbank assets).[11] The relation between loans and total assets is then given by the Leontief inverse $\mathbf{B} = (\mathbf{I} - \mathbf{A})^{-1}$ (Leontief, 1936), which captures all direct and indirect connections between banks:

$$\mathbf{q} = (\mathbf{I} - \mathbf{A})^{-1}\mathbf{l} = \mathbf{B}\mathbf{l}. \tag{3}$$

A useful property of the Leontief inverse is that it can be rewritten as

$$\mathbf{B} = (\mathbf{I} - \mathbf{A})^{-1} = \sum_{i=0}^{\infty} \mathbf{A}^i = \mathbf{I} + \mathbf{A} + \mathbf{A}\mathbf{A} + \dots \tag{4}$$

such that we can rewrite Equation 3 as an infinite sum series

$$\mathbf{q} = \mathbf{l} + \mathbf{A}\mathbf{l} + \mathbf{A}\mathbf{A}\mathbf{l} + \dots. \tag{5}$$

the vector $\mathbf{x}$ on its main diagonal.

[11] See Miller and Blair (2009) for a comprehensive treatment of input-output analysis.

2.2 Shock transmission in the banking system

What happens when a shock hits bank assets in this stylised banking system?[12] We assume that the relative interbank matrix $\mathbf{A} = \mathbf{X}\hat{\mathbf{q}}^{-1}$ remains constant, which implies $\mathbf{A^0} = \mathbf{A^1} = \mathbf{A}$ and $\mathbf{B^0} = \mathbf{B^1} = \mathbf{B}$.[13] For the system's balance sheet equation, this gives $\mathbf{q^0} = \mathbf{Bl^0}$ and $\mathbf{q^1} = \mathbf{Bl^1}$. Letting $\mathbf{\Delta q} = \mathbf{q^1} - \mathbf{q^0}$ and $\mathbf{\Delta l} = \mathbf{l^1} - \mathbf{l^0}$ yields

$$\mathbf{\Delta q} = \mathbf{Bl^1} - \mathbf{Bl^0} = \mathbf{B\Delta l}, \tag{6}$$

which makes it possible to compute how the change in external assets following the shock maps to a change in total assets. The size of the period-1 shock to external assets is therefore defined as $\mathbf{\Delta l}$. The framework accommodates a variety of shock scenarios. The shock can hit the external assets of a single bank, such that only one entry in the vector $\mathbf{l^1}$ changes relative to $\mathbf{l^0}$. This could occur if the bank was performing highly specialized lending to a certain sector that is unable to repay part or all of its loans. The shock can also hit several or all banks' external assets and its size can also be heterogeneous across banks. These cases could occur if there were regional or sector-specific shocks that would affect banks to different degrees.

Recalling Equation 5 we can decompose the shock transmission as:

$$\mathbf{\Delta q} = \mathbf{\Delta l} + \mathbf{A\Delta l} + \mathbf{AA\Delta l} + ... \tag{7}$$

Equation 7 decomposes the effect of a shock into: (i) the initial loss due to the shock (the first term on the right-hand side), (ii) the additional loss due to the direct effect of the shock as transmitted by the holders of the affected assets (second term), and (iii) higher-order terms, i.e. the indirect effects due to the repercussion of the first two effects. There is an amplification of the initial shock $\mathbf{\Delta l}$ via interbank exposures.[14]

It is also possible to read Equation 7 in terms of risk channels. The initial shock to external assets may lead the bank to be insolvent or bring it closer to insolvency by depleting its equity. The reduction in interbank lending captured in the subsequent terms is akin to liquidity risk or rollover risk for the banks who lose funding. The reduction could also be interpreted as default on a fraction of interbank loans; then this would capture counterparty risk.

The framework allows us to deal equally easily with *changes* in, as well as *levels* of, external and

[12] Here and throughout we will use superscripts "0" to represent initial values and "1" for values of variables after the shock to external assets has materialized.

[13] For ease of exposition, we defer the discussion of assumptions to Appendix A. It is however worth mentioning at this stage that this assumption does not imply that the interbank matrix itself remains constant, but rather that the change in interbank exposures is proportional to the shock, and that existing interbank relationships are not terminated nor new relationships added.

[14] There is a formal resemblance between expression Equation 7 and traditional impulse-responses from VAR analyses. In Appendix B we illustrate this with a simple numerical example. While in theory higher order terms can contribute to shock amplification, the evidence from our empirical analysis suggests that in practice this is rather negligible.

total assets. Equation 6 can also be rewritten in terms of levels of the new (i.e. after the shock) external and total assets. Equation 8 computes the new total assets

$$\mathbf{q^1} = \mathbf{Bl^1} = \mathbf{l^1} + \mathbf{Al^1} + \mathbf{AAl^1} + \dots \tag{8}$$

where each term on the right-hand side represents a component of the new total assets.

In what follows we extend this basic stylised framework, in particular in terms of more granularity on the definition of external (i.e. non interbank) assets.

2.3 Extension I: overlapping portfolios

This section modifies the framework by allowing each bank to hold a portfolio of assets. Overlapping portfolios are an indirect channel of contagion, since common exposures will jointly affect financial institutions which are not directly linked by financial exposures between them. Exposures to the same distressed sovereign or to mortgage-backed securities are recent examples where overlapping portfolios have led to simultaneous losses at banks that were not directly linked.

Each bank i holds a portfolio of m assets: w_{ik} is the ratio of asset k held by bank i over the total holdings of asset k by all n banks. We denote by $\mathbf{W}$ the $n \times m$ matrix of these weights and by $\mathbf{h}$ a $m \times 1$ vector with each entry representing the total system holdings of each asset k. In Equation 1 we replace $\mathbf{l}$ by $\mathbf{Wh}$

$$\mathbf{e} + \mathbf{d} + \mathbf{X'i} = \mathbf{Xi} + \mathbf{Wh}. \tag{9}$$

In analogy, Equation 3 becomes:

$$\mathbf{q} = \mathbf{BWh}. \tag{10}$$

The matrix $\mathbf{W}$ now also has an influence on how a shock feeds through to total assets. The framework is again able to accommodate a variety of idiosyncratic and common shocks. In the special case where $\mathbf{h}$ is of length n and the weighting matrix $\mathbf{W}$ is just the identity matrix, we recover the basic setup of Section 2.1.

To sum up, the basic framework can account for contagion due to the direct interbank channel and due to the indirect overlapping portfolio channel. Portfolio overlap is reflected in the entries of matrix $\mathbf{W}$, in particular when read by column. This simple framework can accommodate any level of granularity in the data and it is possible in principle to analyze the propagation of very specific shocks, such as the default of an issuer on an individual security.

2.4 Extension II: prices and fire sales

2.4.1 Introducing prices

In this section, we will introduce prices into the framework, where prices will be relative prices between market and book values. We assume that initially the relative price of each asset is 1. This allows to write the following:

$$\mathbf{l} = \mathbf{W}\mathbf{h} = \mathbf{W}\hat{\mathbf{h}}\mathbf{p}, \tag{11}$$

where we have decomposed vector $\mathbf{h}$ into an $m \times m$ diagonal matrix $\hat{\mathbf{h}}$ times a $m \times 1$ vector of relative prices $\mathbf{p}$.

Throughout the rest of the paper, we will assess the propagation of a shock to the price of assets. These shocks are typically modeled as drops in assets' value, e.g. a drop in the value of sovereign bonds of distressed euro area countries. Due to marking to market of assets, this can be thought of as a shock that hits the asset side of the bank balance sheet. Assuming a change in relative prices from one period to the next, one can then update the system and compute the change in total assets of each bank:

$$\boldsymbol{\Delta}\mathbf{q} = \mathbf{B}\mathbf{W}\hat{\mathbf{h}}\boldsymbol{\Delta}\mathbf{p} \tag{12}$$

Equation 12 maps the change in prices of individual assets held by the banking system into a change of total assets of each bank. It is also possible to quantify the amplification of the shock to common exposures via interbank exposures. Decomposing the shock transmission process in an analogous manner to Equation 7, we get

$$\boldsymbol{\Delta}\mathbf{q} = \mathbf{W}\hat{\mathbf{h}}\boldsymbol{\Delta}\mathbf{p} + \mathbf{A}\mathbf{W}\hat{\mathbf{h}}\boldsymbol{\Delta}\mathbf{p} + \mathbf{A}\mathbf{A}\mathbf{W}\hat{\mathbf{h}}\boldsymbol{\Delta}\mathbf{p} + \ldots, \tag{13}$$

where the first term represents the initial effect of the shock, the second term is the direct effect of the shock on the interbank market and the third and subsequent terms are the indirect feedback effects. Interbank exposures therefore have the effect of amplifying the shock to common assets.

2.4.2 Adding fire sales

In this section we integrate price dynamics via fire sales into the framework. As preliminary step, we need to introduce additional notation to account for the split between assets held in the trading book and in the banking book.

We split external assets into those in the trading and banking books. We denote by $\mathbf{l_s}$ a vector of length $n \times 1$, with each entry representing banks' holdings of each asset k in the trading book. Similarly, $\mathbf{l_b}$ denotes an $n \times 1$ vector with each entry representing banks' holdings of each asset k

in the banking book. We thus have $\mathbf{l} = \mathbf{l_b} + \mathbf{l_s}$. We similarly decompose the vector $\mathbf{h}$. Accordingly, $\mathbf{h_s}$ ($\mathbf{h_b}$) is a vector of length $m \times 1$, with each entry representing the total system holdings of each asset k in the trading (banking) book, such that $\mathbf{h} = \mathbf{h_b} + \mathbf{h_s}$.

We also introduce two weighting matrices for the trading book, since this is where the shock hits and where the subsequent losses from fire sales materialise. The first matrix is defined by holdings relative to all banks. Each bank i holds a portfolio of m assets: w_{sik} is the ratio of asset k held by bank i in its trading book over the total holdings of asset k by all n banks in their trading books. $\mathbf{W_s}$ is the $n \times m$ matrix of these weights.

The second matrix is defined relative to total trading book assets of bank i. We denote it by $\mathbf{Z_s}$, with element z_{sik} capturing the ratio of asset k held by bank i in its trading book over total holdings of bank i in its trading book. With this notation, we have the following two equations:

$$\mathbf{l_s} = \mathbf{W_s}\mathbf{h_s}, \tag{14}$$

$$\mathbf{h_s} = \mathbf{l_s}\mathbf{Z_s}. \tag{15}$$

The analysis of fire sales assumes there are three periods, indexed by 0, 1 ad 2. Initial external assets held in the trading book in period 0 are given by $\mathbf{l_s^0} = \mathbf{W_s}\hat{\mathbf{h}}_\mathbf{s}^\mathbf{0}\mathbf{p^0}$. In period 1 a shock to asset values occurs which leads to an updated vector $\mathbf{p^1}$. We integrate price dynamics by building on the fire sales model developed by Greenwood et al. (2015) and expanded by Duarte and Eisenbach (2020), to which we add the distinction between the banking book and the trading book, the computation of the period 2 price and of the impact of the asset sales on the balance sheet of banks.

Fire sales take place between periods 1 and 2. The detailed computational steps to calculate the fire sales are the following. In Step 1, an initial exogenous shock occurs. The shock is defined as a shock to asset returns $\mathbf{r^1} = (\mathbf{p^1} - \mathbf{p^0})$ which leads to a change in bank returns of $\mathbf{Z_s}\mathbf{r^1}$ in Step 2.

Banks sell assets to return to leverage targets after the shock occurs. Let $\mathbf{\Lambda}$ be the $n \times n$ diagonal matrix of leverage ratios of debt to equity, where each diagonal element is $\Lambda_{ii} = \frac{q_i^0 - e_i}{e_i}$. This matrix captures the pre-shock leverage positions of all banks, and is used within our framework as the target leverage banks aim for. In Step 3 the shortfall to regain target leverage is computed as:

$$\hat{\mathbf{l}}_\mathbf{s}^\mathbf{0}\mathbf{\Lambda}\mathbf{Z_s}\mathbf{r^1} \text{ for the system } (n \times 1)$$
$$\hat{l}_{sii}^0\Lambda_{ii}\sum_k Z_{sik}r_k^1 \text{ for bank } i$$

Target exposures remain fixed in percentage terms, implying that banks sell assets proportionately to their existing holdings. In Step 4 net asset sales are then given by:

$$\mathbf{Z}_\mathbf{s}'\hat{\mathbf{l}}_\mathbf{s}^\mathbf{0}\mathbf{\Lambda}\mathbf{Z_s}\mathbf{r^1} \text{ for the system } (m \times 1)$$

$\sum_i Z_{sik'} \hat{l}^0_{sii} \Lambda_{ii} \sum_k Z_{sik} r^1_k$ for asset k'

where k' denotes one specific asset out of the m assets in vector $\mathbf{h}$. Let $\mathbf{\Phi}$ be the $m \times 1$ vector of net asset purchases by all banks during period 1. If banks keep their portfolio shares constant, then their net asset purchases are equal to $\mathbf{\Phi} = \mathbf{Z_s'} \mathbf{\hat{l}^0_s} \mathbf{\Lambda} \mathbf{Z_s} \mathbf{r^1}$ (as computed in Step 4 above). Period 2 external assets $\mathbf{h^2}$ are then equal to the initial assets plus their net asset purchases $\mathbf{h^0} + \mathbf{\Phi}$. [15] In Step 5, the final computational step, the price impact of the fire sale is computed according to a linear model, with $\mathbf{\hat{\Theta}}$ denoting an $m \times m$ diagonal matrix with price impact coefficients for each asset:

$$\mathbf{\hat{\Theta}} \mathbf{Z_s'} \mathbf{\hat{l}^0} \mathbf{\Lambda} \mathbf{Z_s} \mathbf{r^1} \text{ for the system} (m \times 1)$$
$$\hat{\Theta}_{k'k'} \sum_i Z_{sik'} \hat{l}^0_{sii} \Lambda_{ii} \sum_k Z_{sik} r^1_k \text{ for asset } k'$$

The price impact is related to the price in period 2 in the following way: $\mathbf{p^2} - \mathbf{p^1} = \mathbf{\hat{\Theta}} \mathbf{\Phi}$. The new price vector is then given by $\mathbf{p^2} = \mathbf{\hat{\Theta}} \mathbf{\Phi} + \mathbf{p^1}$. The price effect of fire sales crucially depends on matrix $\mathbf{\hat{\Theta}}$. Calibration of this matrix is thus important, as we will discuss later.

Since the initial shock only affects the assets held in the trading book, period one external assets are computed as the following sum

$$\mathbf{l^1} = \mathbf{l_b} + \mathbf{\hat{l}_s} \mathbf{Z_s} \mathbf{p^1} \tag{16}$$

Linking back to the framework developed above, we now get a more granular view of how shocks and fire sales feed through to external assets (equation 17) and total assets (equation 18):

$$\mathbf{l^2} = \mathbf{l_b} + \mathbf{Z_s} (\mathbf{\hat{h}_s} + \mathbf{\hat{\Phi}}) \mathbf{p^2} \tag{17}$$

$$\begin{aligned}
\mathbf{q^2} &= \mathbf{B} \mathbf{l^2} \\
&= \mathbf{B} (\mathbf{l_b} + \mathbf{l^2_s}) \\
&= \mathbf{BW} (\mathbf{l_b} + \mathbf{Z_s} (\mathbf{\hat{h}_s} + \mathbf{\hat{\Phi}}) \mathbf{p^2}) \\
&= \mathbf{BW} (\mathbf{l_b} + \mathbf{Z_s} (\mathbf{\hat{h}_s} + \mathbf{\hat{\Phi}}) (\mathbf{\hat{\Theta}} \mathbf{\Phi} + \mathbf{p^1}))
\end{aligned} \tag{18}$$

which provides one with the ability to analyse the interaction and amplification of direct and common exposures across banks. It is then possible to trace the impact of an initial shock over several contagion rounds as given by

[15] Note that in order to compute $\mathbf{\Phi}$, we need to make sure that banks do not sell more assets than they have available. When implementing the framework, we therefore use the following definition: $\mathbf{\Phi} = \mathbf{Z_s}' \mathbf{\hat{l}_s} \max(\mathbf{\Lambda} \mathbf{Z_s} \mathbf{r^1}, \mathbf{-1} - \mathbf{Z_s} \mathbf{r_1})$

$$\mathbf{q^2} = \mathbf{l^2} + \mathbf{Al^2} + \mathbf{AAl^2} + \dots \tag{19}$$

Finally, it is also possible to decompose the shock transmission in terms of changes:

$$\mathbf{\Delta q} = \mathbf{\Delta l} + \mathbf{A\Delta l} + \mathbf{AA\Delta l} + \dots, \tag{20}$$

where $\mathbf{\Delta q} = \mathbf{q^2} - \mathbf{q^0}$ and $\mathbf{\Delta l} = \mathbf{l^2} - \mathbf{l^0}$. Equation 20 can then be compared to an analogous decomposition without price dynamics as presented in Equation 13.

Assumptions in fire sales extension. The fire sale extension is based on five assumptions, which we review briefly below. For an extensive discussion of the assumptions, we refer the reader to Greenwood et al. (2015) and Duarte and Eisenbach (2020).

Assumption #1: banks target leverage. Empirical evidence for such behavior can be found in Adrian and Shin (2010, 2011). This assumption gets further support with the introduction of the Basel leverage ratio, which has become a binding capital requirement for banks in January 2018.

Assumption #2: banks retrace the increase in leverage by selling assets and not by raising equity. One of the reasons for such a behavior is that systemic risk is usually accompanied by a general deterioration in equity capital positions, broad distress in capital markets and weak macroeconomic conditions. Raising equity can also be difficult or undesirable for economic or signaling reasons (Shleifer and Vishny (1992)).

Assumption #3: banks sell assets proportionally to their initial holdings. The computational advantage is that proportional liquidation keeps the framework linear. This assumption also has the benefit that it allows us to be agnostic about liquidation order, for example regarding the decision of selling liquid or illiquid assets first. Selling liquid assets first minimizes price impact, but selling illiquid assets first may be desirable under regulatory requirements based on risk-weighted assets (Cifuentes et al., 2005), since they tend to have the largest risk weight (Cont and Schaanning (2017)).

Assumption #4: the price impact of selling assets is proportional to the amount sold. This assumption is in line with intuition, the literature, and fits the patterns of the data well (Ellul et al., 2011; Feldhütter, 2011; Merrill et al., 2012).

Assumption #5: banks stop selling after the first round of fire sales. This is admittedly a stronger assumption. However, Greenwood et al. (2015) and Duarte and Eisenbach (2020) show that the first round captures almost all fire sale spillovers. Against this background, considering higher rounds comes at a computational cost that seems unjustified in terms of additional insights.

2.5 Summary of stress-testing framework

Table 1 summarizes how to compute final period total assets for different combinations of contagion channels. The measures presented in the first column do not include direct interbank exposures as a contagion channel, whereas the second column does, as indicated by the presence of the Leontief inverse $\mathbf{B}$. The measures presented in the first row do not include the effect of price dynamics due to fire sales, but only the impact of the initial shock to asset values ($\mathbf{p^1} - \mathbf{p^0}$). The measures presented in the second row do include price dynamics, as they include the updated fire sale price ($\mathbf{p^2}$) as well as updated asset holdings after fire sales ($\mathbf{h^2}$).

Table 1: Summary of final period total assets ($\mathbf{q}^{final}$) for different combinations of contagion channels

Contagion channels	*1. No interbank exposures*	*2. With interbank exposures*
a. No price dynamics	$\mathbf{q^1} = \mathbf{W\hat{h}^1p^1}$	$\mathbf{q^1} = \mathbf{BW\hat{h}^1p^1}$
b. With price dynamics	$\mathbf{q^2} = \mathbf{W\hat{h}^2p^2}$	$\mathbf{q^2} = \mathbf{BW\hat{h}^2p^2}$

Note: With price dynamics the adjustment takes only one period, hence $\mathbf{q}^{final} = \mathbf{q^1}$. With price dynamics, as discussed above, the shock transmission process culminates in period 2, hence $\mathbf{q}^{final} = \mathbf{q^2}$.

3 Impact and vulnerability measures

Greenwood et al. (2015) define several measures to quantify vulnerability of the banking sector to fire sales. We translate these measures to our setting. Aggregate vulnerability (AV) is defined as the percentage of aggregate bank equity that would be wiped out if banks fire sell in response to a shock to asset values, *excluding* the initial impact of the shock:[16]

$$\mathbf{AV} = \frac{\mathbf{i'\hat{l}_s Z_s \Theta \Phi}}{\mathbf{i'e}}. \tag{21}$$

The contribution that each bank has, through fire sales spillovers, to the aggregate vulnerability of the banking system is referred to as the systemicness (S_j) of bank j. In order to compute this, it is assumed that the shock only affects bank j. Therefore one needs to filter the impact of the shock by pre-multiplying it by $\mathbf{i_j i_j}'$ when computing S_j, such that

$$S_j = \frac{\mathbf{i'\hat{l}_s Z_s \Theta Z_s' \hat{l}_s} i_j i_j' \max(\mathbf{\Lambda Z_s r^1}, \mathbf{-1} - \mathbf{Z_s r_1})}{\mathbf{i'e_j}}. \tag{22}$$

Since systemicness S_j is the contribution of each bank through fire sales spillovers and/or scaling down of interbank exposures to the aggregate vulnerability of the banking system, $AV = \sum_{j=1}^{n} S_j$.

[16] To simplify notation, we use $\mathbf{l_s} = \mathbf{l_s^0}$ in this section.

Indirect vulnerability IV_j measures the fraction of equity of bank j that is wiped out following the deleveraging at other banks and is given by

$$IV_j = \frac{\mathbf{i_j'}\hat{\mathbf{l}}_\mathbf{s}\mathbf{Z_s}\boldsymbol{\Theta}\mathbf{Z_s'}\hat{\mathbf{l}}_\mathbf{s} \max(\boldsymbol{\Lambda}\mathbf{Z_s r_1}, -\mathbf{1} - \mathbf{Z_s r_1})}{e_j}. \tag{23}$$

Direct vulnerability DV_j in turn measures the fraction of equity of bank j that is wiped out following the direct effect of the shock:

$$DV_j = \frac{\mathbf{i_j'}\hat{\mathbf{l}}_\mathbf{s}\mathbf{r^1}}{e_j^0}. \tag{24}$$

3.1 Quantifying the balance sheet impact of deleveraging

The framework developed in the previous section can be used to quantify the balance sheet impact of deleveraging. This is important because it allows us to assess how the endogenous reaction of banks can amplify initial exogenous shocks to asset values and lead to larger losses for the banking system at large. To this end, we build analogous measures to some of the vulnerability indicators in the previous section to quantify this balance sheet impact. Aggregate impact AI measures the balance sheet change when banks deleverage in response to a shock to asset values, excluding the direct impact of the shock $\boldsymbol{\Delta}\mathbf{l} = \mathbf{l^0} - \mathbf{l^1}$. In other words, it singles out the balance sheet adjustment that is only due to deleveraging, encompassing both losses due to price change and changes in holdings due to asset sales.[17] This measure allows us to quantify by how much the banking system's balance sheet shrinks as a result of fire sales. Results are stated in terms of total system assets to provide intuition on the magnitude of the effect. Aggregate impact AI is defined as

$$AI = \frac{\mathbf{i}'(\boldsymbol{\Delta}\mathbf{q}^{final} - \boldsymbol{\Delta}\mathbf{l})}{\mathbf{i}'\mathbf{q^0}}, \tag{25}$$

where $(\boldsymbol{\Delta}\mathbf{q}^{final} - \boldsymbol{\Delta}\mathbf{l})$ is defined in Table 2 for each case. Also note that $\boldsymbol{\Delta}\mathbf{q}^{final}$ is equal to the difference between final period total assets $\mathbf{q}^{final}$ and initial period total assets $\mathbf{q^0}$, where the definition of final period total assets for each case can be found in Table 2. Equally note that for case 1.a AI is zero, since there is no amplification of the shock via fire sales or interbank exposures.

The Indirect Impact II_j measures the balance sheet adjustment of bank j following the deleveraging at other banks and is given by

$$II_j = \frac{i_j'(\boldsymbol{\Delta}\mathbf{q}^{final} - \boldsymbol{\Delta}\mathbf{l})}{q_j^0}. \tag{26}$$

The precise meaning of II_j depends again on which case in Table 2 one considers.

[17] The aggregate vulnerability measure by Greenwood et al. (2015) only captures the losses due to the price impact.

Table 2: Definitions of $\mathbf{\Delta q}^{final} - \mathbf{\Delta l}$ for computing AI for different combinations of contagion channels

Contagion channels	*1. No interbank exposures*	*2. With interbank exposures*
a. No price dynamics	$\mathbf{0}$	$(\mathbf{B}-\mathbf{I})(\mathbf{l^0}-\mathbf{l^1})$
b. With price dynamics	$\mathbf{l^1}-\mathbf{l^2}$	$\mathbf{B}(\mathbf{l^0}-\mathbf{l^2})-(\mathbf{l^0}-\mathbf{l^1})$

The Direct Impact DI_j measures the balance sheet adjustment of bank j following the direct effect of the shock:

$$DI_j = \frac{\mathbf{i_j'\Delta l}}{q_j^0}. \tag{27}$$

Since DI_j does not depend on the definition of $\mathbf{q}^{final}$, it is the same for the four cases presented in Table 2.

4 Interconnectedness of large euro area banks

4.1 Data

The balance sheet data we use is extracted from ECB supervisory statistics and refers to 26 large euro area banking groups.[18] More specifically, we use the quarterly reporting of financial information (FINREP).[19] To build the network of bilateral exposures of banks we use the quarterly large exposure data from the ECB supervisory statistics[20]. To reconstruct the overlapping portfolios of loans and derivatives of each of the 26 banking groups we also use the supervisory statistics.

In order to construct the overlapping portfolios of securities, we make use of two micro-financial datasets, namely the European System of Central Banks' (ESCB) Securities Holdings Statistics by Group (SHSG) and the ECB Centralised Securities Database (CSDB). The SHSG contains quarterly data on security-by-security holdings of debt securities, listed equity and fund shares, covering the

[18]These data are based on the Implementing Technical Standards (ITS) defined by the European Banking Authority which set out a uniform reporting framework for credit institutions domiciled in EU countries rendering the information reported comparable across entities.

[19]To reconstruct the banks' trading book we use the assets reported in the FINREP templates 4.01 (assets held for trading) and 4.03 (available-for-sale financial assets).

[20]The BCBS, in April 2014, introduced a new standard with the aim of ensuring that internationally active banks' exposures to single counterparties are appropriately monitored and limited. The large exposures framework sets prudent limits to large exposures, whereby a large exposure is defined as the sum of all exposures of a bank to a single counterparty that are equal to or above 10% of its Tier 1 capital. The limit is set at 25% of Tier 1 capital. However, in the case of exposure of a global systemically important bank (G-SIB) to another G-SIB, a more stringent limit of 15% of Tier 1 capital applies. Furthermore, a bank must report its 20 largest exposures even if they do not satisfy the definition of a large exposure. All exposures as defined under the risk-based capital framework are subject to the large exposures framework (both banking book and trading book exposures, including derivatives.)

largest 26 euro area banking groups by total assets[21] (i.e. holder-by-holder information). These 26 banking groups represent 59 percent of total euro area banking sector assets. The CSDB is an individual security reference database having detailed information at a monthly frequency on the issuer and the issuance characteristics for the above-mentioned financial instruments. From the SHSG data we can identify all the holdings of debt securities and quoted shares reported by the 26 banking groups, which means that the securities portfolio of each of these banking groups is available at the ISIN-level. In that way we can compute the overlapping portfolios of the banks in the sample, classifying the securities by instrument class (listed shares, short-term debt, long-term debt) and issuing sector.

For the entire empirical application, we use data for the first quarter of 2016.

4.2 Calibration of price impact

We follow Duarte and Eisenbach (2020) in the calibration of the price impact matrix. In the spirit of Shleifer and Vishny (1992), they assume that the price impact of asset k is proportional to its illiquidity and inversely proportional to the wealth of potential buyers of fire-sold assets. Aggregate sales of asset k therefore have price impact given by a measure of illiquidity weighted by the wealth of potential buyers. As in Duarte and Eisenbach (2020), we use the net stable funding ratio (NSFR) haircut on government bonds as a baseline and use it to scale all the other haircuts when computing the price impact. We then weight it by the wealth of potential buyers.

Table 3 lists the NSFR haircuts (our measure of illiquidity) and the resulting price impact. It captures our baseline calibration of price impact. The wealth of potential investors was downloaded from the ECB's statistical data warehouse as the total financial assets of Euro area financial corporations for Q1 2016 and is equal to over 72 trillion euro.[22]

4.3 Descriptive statistics

Banks' balance sheets. Table 4 presents a summary of the main balance sheet items for the 26 large banking groups in our sample. On average, 62 percent of banks' assets are loans, followed by securities holdings with 15.5 percent and derivatives exposures of almost 10 percent. On the liability side, almost 60 percent are accounted for by deposits, whereas debt securities issued represent on

[21] The selection of the banking groups included in SHSG is subject to a Governing Council decision, which is taken at least once a year (the groups are then called reporting banking groups, or shortly RBGs). The SHS Regulation indicates the use of a quantitative threshold (0.5% of consolidated balance sheet of the EU banking Groups), combined with other quantitative and/or qualitative criteria (e.g. to keep certain groups in the sample even if they fall below the threshold over time), to identify banking groups of particular relevance for monetary policy, financial stability or other ESCB tasks. Banking Groups are (parent) credit institutions and all their financial subsidiaries or branches, other than insurance undertakings which have received official authorization in accordance with Art. 6 of Directive 73/239/EEC or Art. 4 of Directive 2002/83/EC.

[22] ECB SDW series key $QSA.Q.N.I8.W0.S12.S1.N.A.LE.F.\_Z.\_Z.XDC.\_T.S.V.N.\_T$ (Total financial assets of Financial corporations of Euro area 19).

Table 3: Calibration of price impact

Issuer	**Security type**	**NSFR haircut**	**Price impact**	**Source**
NFCs	Equity	55%	1.50×10^{-15}	Section II.B, paragraphs 40 and 43
NFCs	ST debt	35%	9.60×10^{-16}	Section II.B, paragraphs 39 and 40
NFCs	LT debt	35%	9.60×10^{-16}	Section II.B, paragraphs 39 and 40
MFIs	Equity	55%	1.50×10^{-15}	Section II.B, paragraphs 40 and 43
MFIs	ST debt	85%	2.33×10^{-15}	Section II.B, paragraph 42
MFIs	LT debt	85%	2.33×10^{-15}	Section II.B, paragraph 42
MMFs	Fund shares	55%	1.50×10^{-15}	Section II.B, paragraphs 40 and 43
IFs	Fund shares	55%	1.50×10^{-15}	Section II.B, paragraphs 40 and 43
IFs	Equity	55%	1.50×10^{-15}	Section II.B, paragraphs 40 and 43
IFs	LT debt	85%	2.33×10^{-15}	Section II.B, paragraph 42
OFIs	Equity	55%	1.50×10^{-15}	Section II.B, paragraphs 40 and 43
OFIs	ST debt	85%	2.33×10^{-15}	Section II.B, paragraph 42
OFIs	LT debt	85%	2.33×10^{-15}	Section II.B, paragraph 42
ICs	Equity	55%	1.50×10^{-15}	Section II.B, paragraphs 40 and 43
ICs	ST debt	35%	9.60×10^{-15}	Section II.B, paragraphs 39 and 40
ICs	LT debt	35%	9.60×10^{-15}	Section II.B, paragraphs 39 and 40
Government	ST debt	5%	1.37×10^{-16}	Section II.B, paragraph 37
Government	LT debt	5%	1.37×10^{-16}	Section II.B, paragraph 37
RoW		100%	2.74×10^{-15}	
Loans to CB		No haircut	0	EU REG 575/2013, Article 416 1.a
Loans to Gov		50%	1.37×10^{-15}	Section II.B, paragraph 40
Loans to MFIs		50%	1.37×10^{-15}	Section II.B, paragraph 40
Loans to OFIs		50%	1.37×10^{-15}	Section II.B, paragraph 40
Loans to NFCs		50%	1.37×10^{-15}	Section II.B, paragraph 40
Loans to HHs		60%	1.64×10^{-15}	Section II.B, paragraph 40-41
Cash		No haircut	0	EU REG 575/2013, Article 416 1.a
Derivatives		100%	2.74×10^{-15}	Section II.B, paragraph 43

Notes: NFC refers to non-financial corporates, MFI to monetary financial institutions, MMF to money market funds, IF to non-MMF investment funds, IC to insurance companies, OFI to other financial institutions, CB to central bank, RoW to rest of the world, and HH to households. Unless otherwise specified, references are made to the NSFR publication https://www.bis.org/bcbs/publ/d295.pdf.

average almost 20 percent of total assets. On average, the banks in the sample hold 4.7 percent of Common Equity Tier 1 (CET1) capital out of total assets.

Table 5 displays additional summary statistics based on balance sheet data from ECB supervisory data. Banks have an average risk-weighted Basel CET1 capital ratio (equity over risk-weighted total assets) of 13.2 percent and a Basel leverage ratio (equity over unweighted total assets) of 5.01 percent on average. The leverage measure used for the computation of the fire sales presented in Section 2.4.2, calculated as debt over equity, is 19.95 on average.

Table 4: Balance sheet items for the 26 banking groups (average in % of total assets)

Assets		**Liabilities**	
Securities holdings	15.5	Debt securities issued	19.5
Loans	62.2	Deposits	59.7
Interbank exposures	2.5	Interbank liabilities	2.03
Derivatives	9.8	Tier 2 capital	1
Cash	4.4	Additional Tier 1 capital	0.3
		Common Equity Tier 1 capital	4.7

Notes: Interbank exposures/liabilities refer to the exposures within the network of the 26 largest euro area banking groups and are based on the supervisory data collected under the large exposure regime. The securities holdings are based on the SHS data and the other balance sheet items are based on supervisory data (FINREP). The percentages are out of reported total assets from the supervisory data and do not sum to 100%.

Table 5: Descriptive statistics of the distribution of total assets, leverage and prudential requirements for the 26 largest euro area banking groups

	System	**p10**	**Med.**	**Mean**	**p90**
Assets (euro billions)	15,413	163	442	592	1,293
Leverage (debt over equity, ratio)	19.63	13.45	19.5	19.95	26.43
Basel leverage ratio (in percent)	4.84	3.64	4.87	5.01	6.91
Basel CET1 ratio (in percent)	12.5	11.1	12.9	13.2	15.8

Banks' portfolios of assets. Table 6 presents descriptive statistics for the banks' portfolios of assets, in other words the distribution of w_{ik} in the data.[23] The three largest asset classes are loans to non-financial corporations, loans to households and derivatives. Holdings of long term government debt also represent a relatively large share at the system level. Some individual banks can also have large exposures to non-financial corporations and other financial institutions through holdings of both short and long term debt as well as equity or shares.

Table 7 presents aggregate figures for the banks' assets available for trading. Tradable assets of the 26 largest euro area banking groups make up about a third of their total assets, or around 5.4 billion euro are the system level. The average banking group in our sample holds 173 billion euro in tradable assets, representing about a quarter of its total assets. However, the distribution is skewed: a bank in the 90^{th} percentile of the distribution holds 437 billion euro, accounting for almost 40% of its total assets.

Table 8 presents descriptive statistics for the banks' portfolios of assets available for trading. The system shares were computed as nominal amount of tradable assets in one asset category over total system assets. For the other descriptive statistics we computed the ratio of holdings in the trading book of one asset category by one bank out of the total holdings in the trading book of

[23] See Section 2.3 for a definition of the weighting matrix $\mathbf{W}$.

Table 6: Banks' portfolio of assets (distribution of w_{ik} in the data), % of total assets

Sector	Instrument	System	p25	Med.	p75
Non-financial corporations (S.11)	**Equity**	1.41	0.01	0.15	0.98
	ST debt	0.04	0	0	0.03
	LT debt	0.51	0.15	0.35	0.77
Credit institutions (S.122)	**Equity**	0.20	0	0.02	0.24
	ST debt	0.07	0	0.03	0.08
	LT debt	1.43	0.90	1.36	1.97
Money market funds (S.123)	**Fund shares**	0.01	0	0	0.01
Non-MMF investment funds (S.124)	**Fund shares**	0.45	0.03	0.16	0.70
Other financial corporations (S.125,S.126,S.127)	**Equity**	0.14	0	0.02	0.12
	ST debt	0.08	0	0.01	0.06
	LT debt	2.33	1.29	1.92	3.04
Insurance companies(S.128)	**Equity**	0.05	0	0	0.07
	LT debt	0.01	0	0	0.02
Government (S.13)	**ST debt**	0.64	0.06	0.28	0.79
	LT debt	8.58	6.15	8.39	11.87
Rest of the world	**All instruments**	0.09	0	0	0.04
Loans to central banks		0.72	0	0.34	1.28
Loans to governments		3.781.89	3.27	6.84	
Loans to credit institutions		6.22	3.22	5.33	9.15
Loans to other financial corporations		6.63	2.98	4.31	8.95
Loans to non-financial corporations		21.58	18.48	22.04	32.25
Loans to households		24.89	14.90	24.10	32.02
Cash		5.94	2.23	4.05	7.36
Derivatives		14.09	6.792	7.89	13.09

Note: The system shares were computed as nominal amount in one asset category over total system assets. The data for the debt and equity instruments is taken from the SHS data and the data on loans, cash and derivatives comes from the ECB supervisory data. p25, Med. and p75 represent respectively the 25^{th} percentile, median and 75^{th} percentile.

Table 7: Descriptive statistics for the banks' assets available for trading

	System	p10	Med.	Mean	p90
Tradable assets (euro millions)	5,361.88	32.82	98.44	172.61	437.05
Tradable assets out of total assets (percent)	34.78	15.58	21.38	24.51	39.46

that asset category by all banks. The five largest asset categories by system share are derivatives, long-term government bonds, loans to other financial corporations, loans to banks and long-term bonds issued by other financial corporations. Our framework also lends itself to assess shocks to individual asset classes and results for a stress scenario for these five categories can be found in Figure 10a. We do not observe a strong relationship between trading assets and leverage for our sample of banks.[24]

[24]The banking groups with the largest leverage in our sample do show a large amount of trading assets. Besides these couple of outliers, however, the relationship is essentially non-existent. For confidentiality reasons we are not able to show the corresponding figures.

Table 8: Descriptive statistics for the bank's portfolio of assets available for trading, in percent

Sector	*Instrument*	System	p25	Med.	p75
Non-financial corporations (S.11)	**Equity**	0.19	0	0	0.06
	ST debt	0.07	0	0	0.05
	LT debt	1.04	0.38	0.71	1.42
Credit institutions (S.122)	**Equity**	0.56	0	0.11	0.54
	ST debt	0.19	0	0.09	0.26
	LT debt	3.54	1.52	3.95	6.07
Money market funds (S.123)	**Fund shares**	0.03	0	0	0.02
Non-MMF investment funds (S.124)	**Fund shares**	1.27	0.12	0.57	1.62
Other financial corporations (S.125,S.126,S.127)	**Equity**	0.42	0	0.10	0.43
	ST debt	0.19	0	0.03	0.12
	LT debt	4.51	2.74	3.53	7.50
Insurance companies(S.128)	**Equity**	0.15	0	0.03	0.19
	LT debt	0.02	0	0	0.03
Government (S.13)	**ST debt**	1.62	0.21	0.97	2.04
	LT debt	21.33	16.94	27.08	39.07
Rest of the world	**All instruments**	0.12	0	0	0
Loans to central banks		0.17	0	0	0.04
Loans to government		0.16	0	0	0.19
Loans to credit institutions		4.59	0	0.47	2.64
Loans to other financial corporations		5.73	0	0.03	2.83
Loans to non-financial corporations		0.49	0	0.05	0.27
Loans to households		0.01	0	0	0.01
Cash		16.77	6.11	14.65	23.17
Derivatives		36.71	22.19	28.33	33.06

Note: The system shares were computed as nominal amount of tradable assets in one asset category over total system assets held in the trading book.

Direct exposure network. Table 9 provides an overview of key topological measures for the interbank network based on the large exposure data.[25] The network is dense: 66 percent of all the possible links exist. This is not surprising, given that the banks in the sample essentially constitute the core of the euro area interbank network. Accordingly, on average any bank in the network has direct connections with 16 other banks. Each bank is on average 1.8, and at most 4, links away.

5 Empirical application

We focus on bank distress (as opposed to default) to keep the framework linear and intuitive. We thus need to make sure that no bank defaults after the initial shock. We set a failure threshold in accordance with the European regulatory environment.

[25] *Average shortest path length* denotes the number of connections in a shortest path linking any two pair of nodes. *Average degree* denotes the number of links every node has to all other nodes, averaged over all nodes in the network. The average degree computed here is the average of the total degree (the sum of the in- and the out-degree, i.e. incoming and outgoing connections). The *density* of the network is the number of existing connections over the number of all possible connections. The *diameter* is the longest of all the calculated shortest paths in a network.

Table 9: Topological measures of the interbank network

Topological measure	Value
Average shortest path length	1.86
Average Degree	16.61
Density	0.66
Diameter	4

EU legislation does not provide for quantitative thresholds to determine whether a bank is failing. Instead, such determination is left to the supervisor or resolution authority. For our model, a benchmark is needed to assess at which capital level a bank would be considered to be failing. One possible threshold would be the minimum requirement of a 4.5 percent CET1, reflecting that buffers and other capital to meet minimum (Pillar 1) and additional supervisory (Pillar 2) capital requirements are depleted.[26] A more conservative assumption would be that a bank is determined to be failing when it has depleted its buffers and half of its Pillar 2 capital add-on. This is the failure threshold we adopt in the simulations below. Based on the average CET1 requirement set by the Single Supervisory Mechanism (SSM), this would put the threshold at 7% CET1.[27]

5.1 Results

We apply a shock of 5 percent to the value of the assets held in the trading book. Since the trading book needs to be marked to market daily, that is where losses will materialize instantly and lead to the write-down of equity to absorb the losses. The shock size in the baseline scenario is chosen based on the maximum value of peak accumulated losses for G-SIBs during the Great Financial Crisis (see Financial Stability Board (2015)).[28]

In the aftermath of a shock to tradable assets, banks suffer aggregate losses of about 30% of initial equity (see Table 10).[29] There is significant variation at the bank level. At the lower end of the distribution (25^{th} percentile) this loss amounts to 14% of initial equity, whereas at the upper end of the distribution (75^{th} percentile) it reaches 32%. Asset sales, which by design only occur

[26] Part Two of the Capital Requirements Regulation (CRR) establishes the own funds requirements (Pillar 1 capital requirements) with which institutions are required to comply. In accordance with Article 104(1)(a) of the Capital Requirements Directive (CRD), Member States must ensure that competent authorities are empowered, inter alia, to require institutions to hold additional own funds requirements (Pillar 2 capital requirements) on a case-by-case basis.

[27] To be precise, this would put the threshold at 4.5%+1/2*(9.9%-4.5%) = 7.2% CET1, which we round to 7%. The 9.9% refers to the average of the CET1 requirements for the euro area banks under direct SSM supervision (the so-called Significant Institutions) published in the Supervisory Review and Evaluation Process (SREP), see the SSM's SREP methodology for 2015 under `https://www.bankingsupervision.europa.eu/ecb/pub/pdf/ssm_srep_methodology_booklet.en.pdf`.

[28] Strictly speaking, the accumulated losses peak at 4.7 percent of total assets (see Table 1 in Financial Stability Board (2015)), which we round to 5 percent. To put this into perspective, when Bear Sterns and Lehman Brothers are included in the sample, the maximum value of peak accumulated losses rises to 9.7%.

[29] As discussed above, the shock does not lead to default at any of the 26 banks in the sample.

within the trading book, can be substantial: slightly over 80% of tradable assets are sold to regain target leverage following a 5% shocks to the price of tradable assets.

Table 10: Losses, asset sales and Basel leverage ratio after a 5% shock to tradable assets

	Aggregate	p25	Med.	Mean	p75
Loss					
As share of initial equity (Direct Vulnerability)	0.304	0.144	0.249	0.275	0.319
As share of initial total assets (Direct Impact)	0.015	0.008	0.011	0.013	0.015
Asset sales					
As share of tradable assets	0.843	0.822	0.833	0.834	0.849
As share of initial total assets	0.293	0.175	0.199	0.243	0.295
Basel Leverage Ratio					
Starting value	4.846	4.216	4.878	5.018	5.908
After initial shock	3.449	3.018	3.660	3.846	4.915
After fire sales	4.848	3.914	4.749	4.940	5.956

In the fire sales framework presented in Section 2.4.2 banks are prompted to sell assets after a shock because they want to restore their initial target leverage levels (calculated as debt over equity). Banks' aggregate initial leverage ratio is 19.6, and increases to 27.9 after the shock hits. After banks sell assets from their trading book, the aggregate leverage (debt-to-equity) ratio goes back to 19.6, i.e. it is fully restored to initial levels. We also present the evolution of the regulatory Basel leverage ratio (capital over total assets) in more detail in Table 10. Before the shock, it stands at 5.0 (4.9) for the average (median) bank. After the initial shock (i.e. before banks' behavioural response to regain target leverage), leverage increases substantially: the ratio goes to 3.8 (3.7) for the average (median) bank. The decline is larger for those banks that are more leveraged to begin with, namely those on the 25^{th} percentile. We see that the descriptive statistics of the initial and the final leverage ratio are very similar (more so for less leveraged banks), showing that banks manage to restore their initial leverage ratio via asset sales from their trading book only.[30]

Sytemicness and vulnerability. The contribution that each bank has, through fire sales spillovers, to the aggregate vulnerability of the banking system is called the systemicness, S_j. The fraction of equity at one bank that is wiped out following the deleveraging at other banks is referred to as Indirect Vulnerability (IV).

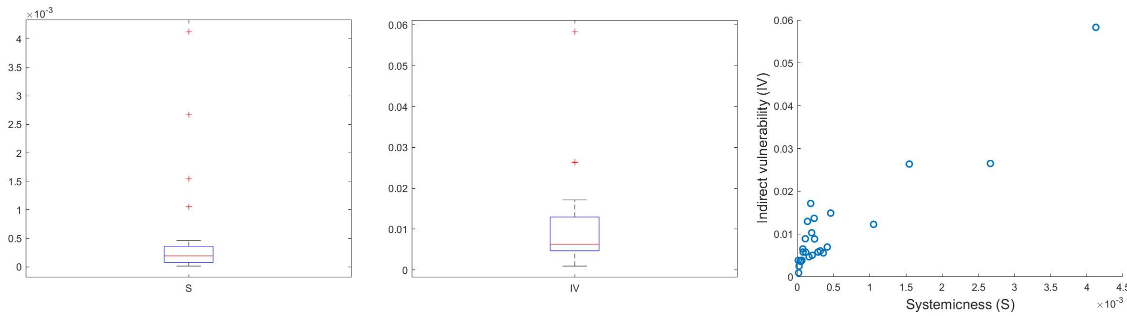
The systemicness of most banks in our sample tends to be quite small. The are a couple of outliers with relatively large contributions to aggregate vulnerability, however (Figure 1a): the contribution of the highest ranking bank based on systemicness is an order of magnitude larger than that of the median bank in the sample. The distribution of Indirect Vulnerability is not as compressed as that of Systemicness, as shown in Figure 1b. The most systemic banks are also the

[30] This result also provides support for the assumption of fire sales lasting for one period, as discussed in Section 2.4.2.

ones with highest indirect vulnerability, which might be a concern for financial stability since it implies that the banks are both contributing most to vulnerability from fire sales and are most vulnerable to losses from fire sales by others (Figure 1c).

Figure 1: Systemicness and Indirect Vulnerability in baseline scenario

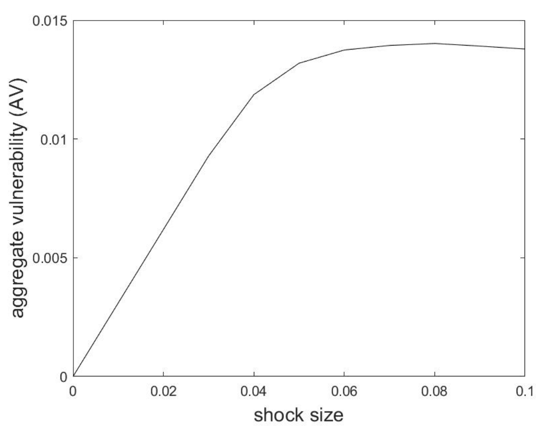
(a) Systemicness (S_j) **(b)** Indirect Vulnerability (IV_j) **(c)** S_j versus IV_j



Notes: The boxes are delimited by the 25th and 75th percentiles of the samples, respectively. The red line is the sample median. Outliers are displayed with a red + sign.

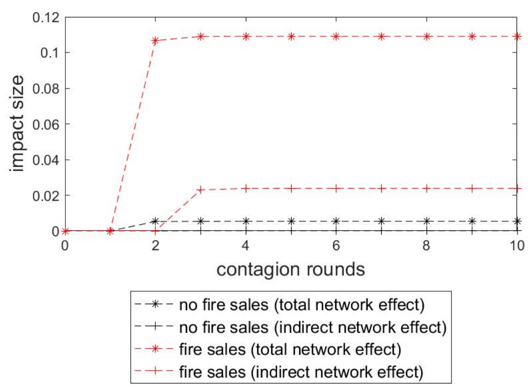
The aggregate vulnerability measure in our benchmark calibration is quite small (0.013), especially when compared to numbers in Greenwood et al. (2015). This implies that following a 5% shock to the price of assets in the trading book, 1.3% of aggregate bank equity is wiped out to fire sales. Figure 2 presents Aggregate Vulnerability as a function of shock size. Aggregate Vulnerability grows linearly until about our benchmark shock size, and then flattens out as larger shock sizes push banks against the limit of what they can sell.

Figure 2: Aggregate vulnerability by shock size (baseline price impact)



Contagion accounting. In order to quantitatively gauge the importance of direct interbank interconnectedness in propagating losses, we compute the loss that is only due to the bank network. We compute the additional loss induced by the network on top of the losses inflicted by the initial shock (the total network effect), which captures loss propagation due to direct links between counterparties. We also compute the losses inflicted by indirect linkages, namely those created on top of the losses due to the shock and the direct links (the indirect network effect). Figure 3 presents the results.

Figure 3: Additional loss induced by the network (ratio to total system equity)



Note: The total network effect is obtained by subtracting the first term (the initial impact of the shock) from all terms in the cumulative version of Equations 13 and then dividing each term by total system equity. For the total network effect for the fire sales case we perform the same computation with Equation 20. The indirect network effect is obtained by subtracting the first and second terms (initial impact of the shock and direct network effect) from all terms in the cumulative version of Equation 13 and then dividing each term by total system equity. For the indirect network effect for the fire sales case we perform the same computation with Equation 20.

Fire sales are the key contributor to contagion. This can also be clearly seen when computing the aggregate impact indicator which captures the balance sheet impact of deleveraging in terms of total initial assets (see 3.1). Table 11 implements empirically the measures in Table 2 for our sample of 26 large European banks. Without interbank exposures nor fire sales, the aggregate impact is naturally zero. When interbank contagion is present but there are no losses due to fire sales, the effect is non zero, but is nonetheless negligible. It is only in the case when fire sale are present that aggregate impact becomes meaningful. When we combine interbank contagion and fire sales, the contribution of the former is also small (though it is larger than when looking at interbank contagion on a standalone basis). While network contagion has in principle the potential to cause large losses (Glasserman and Young (2015a), Duarte and Jones (2017)), in practice its contribution is small and dwarfed by fire sales.

Figures 4a and 4b illustrate this further, where the first figure shows the impact in terms of levels and the second shows the cumulative impact. They depict the impact relative to total banking

Table 11: Accounting for contagion – Aggregate impact (AI) in baseline scenario

	1. No interbank exposures	*2. With interbank exposures*
a. No price dynamics	0	0.00026
b. With price dynamics	0.28097	0.28625

Table 12: Accounting for contagion – Aggregate impact (AI) out of system equity in baseline scenario

	1. No interbank exposures	*2. With interbank exposures*
a. No price dynamics	0	0.0053
b. With price dynamics	5.79	5.90
bottomrule		

Note: Share of initial total assets.

sector equity of the baseline 5 percent decline in prices of assets in the trading book. Round 0 is the initial period and round 1 is when the shock hits.[31] For the case without fire sales, the initial impact of the 5 percent shock leads to a loss of aggregate equity of 30 percent (see first row in Table 10). The impact in further rounds is negligible. For the case with fire sales, the impact in period 1 is 5.9 times aggregate equity (as illustrated in the lower right corner cell of Table 12). This is due to the fact that Equation 20 takes the initial shock to prices as well as resulting fire sales as an input to calculate their joint impact on the interbank market. This does not imply that aggregate bank equity would be wiped out 6 times, as the asset sales do not translate into one for one losses in equity. Expressing our results relative to equity does allow us to compare magnitudes in a common unit. Figure 4b in particular illustrates that amplification through the interbank market is negligible in our empirical illustration.

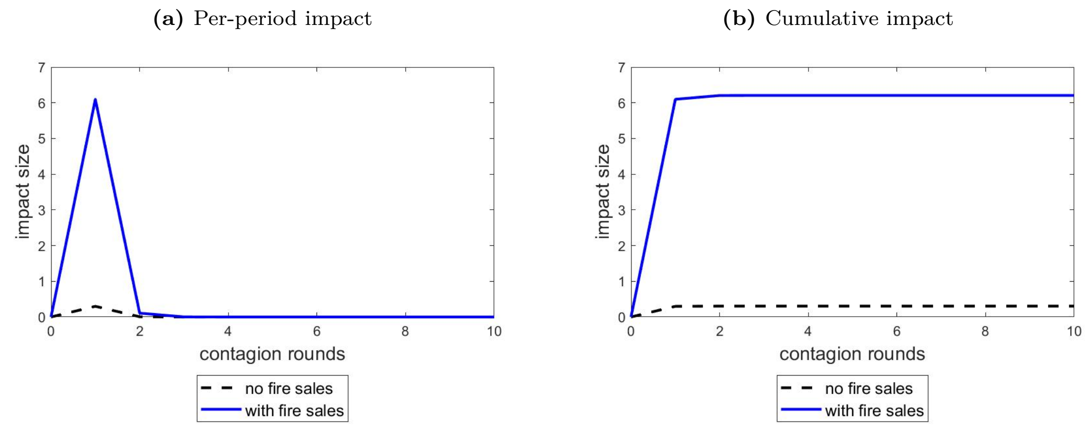
Key drivers: fire sold assets and price impact. There are two main reasons behind the difference in size between the systemicness and indirect vulnerability measures as computed by Greenwood et al. (2015) and the versions we compute.[32] The first is that their shock applies to the whole asset side rather than trading assets (and selling accordingly also involves the whole asset side). The second one is the role of the price impact calibration.

Figures 5a and 5b compare the Systemicness and Indirect Vulnerability measures developed in this paper to the ones developed by Greenwood et al. (2015) for the *same* shock size. Blue lines denote the 45 degree line. The difference in size is clearly illustrated by the fact that there are

[31] For the case without fire sales, each round from periods 1 to 10 corresponds to a term in Equation 7. For the case including fire sales, each round from rounds 1 to 10 corresponds to a term in Equation 20.

[32] In Appendix C we show how to compute our measures without distinguishing between trading book and banking book.

Figure 4: Impact of a 5 percent shock to tradable assets (ratio to total banking sector equity)



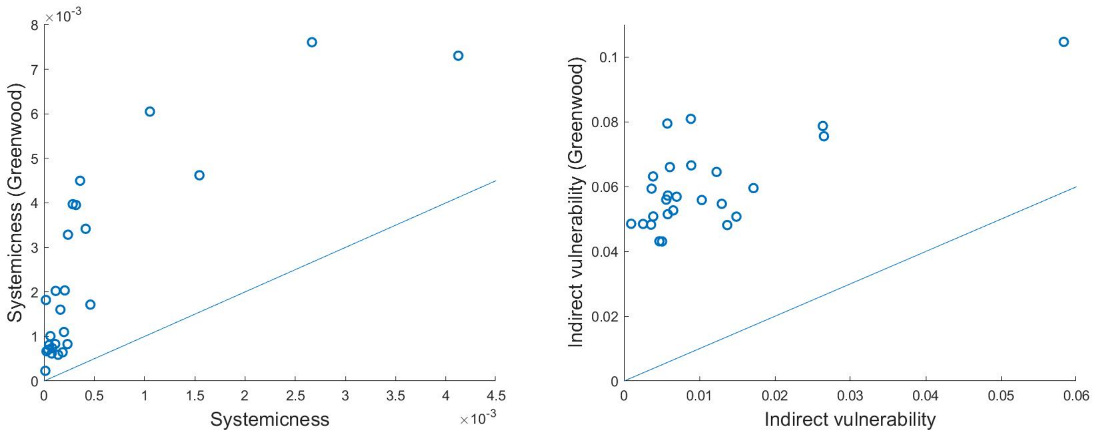
no dots below this line: for any given bank, the measure as computed here is significantly smaller than the corresponding measure as computed by Greenwood et al. (2015). This difference can be explained both by the different calibration of the price impact of fire sales (see below) and by differences in terms of assets subject to fire sales. Our measure of systemicness does a poorer job in distinguishing between low systemicness banks, but does a better in distinguishing those at the top – i.e. those that contribute the most to aggregate vulnerability. A similar pattern is observed for indirect vulnerability.

The calibration of the price impact is also highly consequential. To illustrate this point, Table 13 presents the distribution of systemicness and indirect vulnerability for our baseline calibration, multiplies our baseline price impact by 100 and 1000, and further compares this to the results from applying Greenwood et al. (2015) calibration (namely a constant price impact drawn from Ellul et al. (2011)). It is worth noting that our baseline calibration is based on a more detailed computation of price impact by asset class. That said, for both indirect vulnerability and systemicness, it is only by multiplying our price impact by 1000 that we are able to surpass the results from Greenwood et al. (2015). Similarly, Table 14 presents a similar picture for aggregate impact.

Taken together, these findings highlight the importance of an accurate identification of the assets potentially subject to fire sales, as well as price impact of selling those assets. The large effects found in Greenwood et al. (2015) could thus be driven by a combination of these two issues. Our baseline calibration based on NSFR delivers relatively small effects. The results we present point to the need for further research work that can inform a more accurate calibration of price impact effects of fire sales.

Figure 5: Comparison with equivalent measures from Greenwood et al. (2015)

(a) Systemicness (S_j) **(b)** Indirect Vulnerability (IV_j)



Notes: 45 degree lines in blue.

Table 13: Price impact, systemicness and indirect vulnerability

Scenario	**p25**	**Med.**	**Mean**	**p75**
Systemicness				
Baseline	7.6572e-05	0.00019202	0.00050736	0.00035937
Baseline×100	0.0076572	0.019202	0.050736	0.035937
Baseline×1000	0.076572	0.19202	0.50736	0.35937
Greenwood et al. (2015)	0.0501	0.11127	0.27439	0.23433
Indirect Vulnerability				
Baseline	0.0046842	0.0062928	0.010797	0.012952
Baseline×100	0.46842	0.62928	1.0797	1.2952
Baseline×1000	4.6842	6.2928	10.7972	12.9519
Greenwood et al. (2015)	3.429	4.9898	6.1387	6.8637

Table 14: Aggregate impact (AI) for alternative price impacts (5% shock)

	1. No interbank exposures	**2. With interbank exposures**
Baseline	0.28097	0.28625
Baseline × 100	0.2889	0.29432
Baseline × 1000	0.36097	0.36767
Greenwood et al. (2015)	0.33181	0.338

6 Conclusion

This paper provides a simple and tractable stress-testing framework to assess loss dynamics in the banking sector. We start from a basic accounting scheme based on direct interbank exposures, and expand on it by incorporating overlapping portfolios, prices, and fire sale dynamics based on a leverage targeting adjustment rule. The modular way in which we develop the framework allows us to look at different contagion channels separately to assess which channels matter more empirically.

We apply the framework to several granular data sets for the largest 26 euro area banking groups. We find that a 5 percent shock to prices of assets held in the trading book leads to an initial loss of 30 percent of system equity and an additional loss of 1.3 percent of system equity due to fire sales spillovers. We also compute measures to quantify the balance sheet impact of deleveraging. Deleveraging in the interbank market on its own does not lead to large balance sheet adjustments, which confirms empirically that contagion through direct interbank exposures is a relatively unimportant channel. Furthermore, higher order connections (i.e. beyond direct) through interbank exposures do not meaningfully contribute to shock amplification within our sample. Only in the presence of fire sales dynamics is there a significant balance sheet change due to deleveraging.

Our findings underscore that accounting for contagion should give a prominent role to overlapping portfolios and associated negative price externalities due to fire sales. Importantly, more effort should be directed by the stress-testing community towards the calibration of the price impact of fire sales

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A Discussion of assumptions in the basic framework

In the 2.1 we assumed the *relative* interbank matrix to remain constant, such that $\mathbf{A^0} = \mathbf{A^1} = \mathbf{A}$, which implies $\mathbf{B^0} = \mathbf{B^1} = \mathbf{B}$. This results in $\mathbf{q^0} = \mathbf{Bl^0}$ and $\mathbf{q^1} = \mathbf{Bl^1}$. This is necessary for $\mathbf{\Delta q} = \mathbf{B\Delta l}$ to hold, which is crucial for the clear decomposition of the shock transmission in Equation 7. The assumption that the relative interbank matrix $\mathbf{A}$ stays constant is not very restrictive in practice, since it still allows for interbank exposures to change in response to a shock. To gain intuition, let us write down the elements of Equation 7 for $n = 3$. The change in external assets after the shock is given by $\mathbf{\Delta l} = [\Delta l_1, \Delta l_2, \Delta l_3]'$. The matrix of asset-weighted interbank exposures is

$$\mathbf{A} = \begin{pmatrix} \frac{x_{11}}{q_1^0} & \frac{x_{12}}{q_2^0} & \frac{x_{13}}{q_3^0} \\ \frac{x_{21}}{q_1^0} & \frac{x_{22}}{q_2^0} & \frac{x_{23}}{q_3^0} \\ \frac{x_{31}}{q_1^0} & \frac{x_{32}}{q_2^0} & \frac{x_{33}}{q_3^0} \end{pmatrix} \tag{28}$$

The second term on the right-hand side of Equation 7 is then

$$\mathbf{A\Delta l} = \begin{pmatrix} \frac{x_{11}}{q_1^0}\Delta l_1 + \frac{x_{12}}{q_2^0}\Delta l_2 + \frac{x_{13}}{q_3^0}\Delta l_3 \\ \frac{x_{21}}{q_1^0}\Delta l_1 + \frac{x_{22}}{q_2^0}\Delta l_2 + \frac{x_{23}}{q_3^0}\Delta l_3 \\ \frac{x_{31}}{q_1^0}\Delta l_1 + \frac{x_{32}}{q_2^0}\Delta l_2 + \frac{x_{33}}{q_3^0}\Delta l_3 \end{pmatrix} \tag{29}$$

In the first round of direct spillovers, interbank positions x_{ij} get reduced proportionally to the change in external assets out of total assets $\frac{\Delta l_j}{q_j^0}$ of the borrowing bank j. The use of funds by the borrowing bank j is reduced by the shock to its external assets and therefore it also requires less funding from the interbank market.[33]

It is also possible to quantify how the interbank matrix gets affected in the second contagion round represented by the term $\mathbf{A}^2\mathbf{\Delta l}$, which is where the indirect spillovers kick in. The first entry of the matrix $\mathbf{A}^2\mathbf{\Delta l}$ is computed the following way:

$$\begin{aligned} & (\frac{x_{11}}{q_1^0}\frac{x_{11}}{q_1^0} + \frac{x_{12}}{q_2^0}\frac{x_{21}}{q_1^0} + \frac{x_{13}}{q_3^0}\frac{x_{31}}{q_1^0})\Delta l_1 \\ + \quad & (\frac{x_{11}}{q_1^0}\frac{x_{12}}{q_2^0} + \frac{x_{12}}{q_2^0}\frac{x_{22}}{q_2^0} + \frac{x_{13}}{q_3^0}\frac{x_{32}}{q_2^0})\Delta l_2 \\ + \quad & (\frac{x_{11}}{q_1^0}\frac{x_{13}}{q_3^0} + \frac{x_{12}}{q_2^0}\frac{x_{23}}{q_3^0} + \frac{x_{13}}{q_3^0}\frac{x_{33}}{q_3^0})\Delta l_3 \end{aligned} \tag{30}$$

[33] In terms of the classic input-output model, a negative shock to final demand (here external assets) leads to a reduction in the demand for intermediate inputs (here interbank loans).

The present framework therefore does allow to track the changes in interbank exposures implied by a given shock in a very granular way. In Equation 30 we see for example precisely how cross-terms enter into the computation of $\mathbf{A}^2\boldsymbol{\Delta}\mathbf{l}$, which illustrate how spillovers and feedback loops in the interbank matrix might amplify the initial shock.

In summary, even though we assume the relative interbank matrix $\mathbf{A}$ to be stable to perform our calculations, this does not imply that the end result does not take into account a change in interbank exposures in a crisis situation. What it does imply however is that: (i) the change in interbank exposures is proportional to the shock, and (ii) no links are severed and no links are added in period 1 in response to the shock.

First, the way interbank exposures are updated is fully in line with the standard literature on contagion simulations and stress testing. As highlighted in the survey by Upper (2011), many contagion simulations in interbank networks use exogenously given recovery rates to compute the impact of a shock to external assets. In the present framework recovery rates have an intuitive link to the balance sheet structure of banks, where the extent of the reduction depends on the change in external assets in relation to total assets. This is not an assumption we make, but is inherent in the structure of the framework. The total recovery rate[34] β_j of bank j on its interbank assets that is implied by the input-output framework is given by

$$\beta_j = \frac{\mathbf{i_j'X^1i}}{\mathbf{i_j'X^0i}} = \frac{q_j^1 - l_j^1}{q_j^0 - l_j^0} = \frac{\mathbf{i_j'Bi}l_j^1 - l_j^1}{\mathbf{i_j'Bi}l_j^0 - l_j^0} = \frac{(\mathbf{i_j'Bi} - 1)l_j^1}{(\mathbf{i_j'Bi} - 1)l_j^0}, \tag{31}$$

where we used the fact that Equation 2 is of the same form as Equation 8 in the sense that total assets are equal to interbank assets and external assets:

$$\mathbf{q^1} = \mathbf{l^1} + \underbrace{\mathbf{Al^1} + \mathbf{AAl^1}\dots}_{=\mathbf{X^1i}},$$

which implies $\mathbf{q^1} - \mathbf{l^1} = \mathbf{X^1i}$, where $\mathbf{X^1i}$ is the vector of interbank assets after the shock.

Second, the underlying stability of the interbank matrix $\mathbf{A}$ is not one where the monetary exposures of the banks remain constant, but where the relationships remain stable. Indeed, proportional change in the interbank matrix implies that no links are severed and no links are added in period 1 when the shock hits. If we think of the interbank matrix as an adjacency matrix, the zeros remain zeros and the ones remain ones. The empirical literature on relationship lending finds that interbank relations are remarkably stable even in periods of stress (Abbassi et al., 2014; Affinito, 2012; Bräuning and Fecht, 2016; Cocco et al., 2009). In crisis times, counterparty risk might be elevated, so banks might charge higher interest rates or do not rollover loans in the same quantity. It is

[34]Backing out the recovery rates implied by our framework is useful to compare them to the recovery rates implied by other clearing algorithms such as the conservative clearing algorithm by Eisenberg and Noe (2001) or sequential default algorithms where the loss-given-default rate is ad hoc and exogenously fixed (which is one minus the recovery rate computed here), as for example in Furfine (2003).

therefore plausible that the monetary quantity of the exposures is reduced, which is what happens in the present framework. Since trust is also an important aspect in the interbank market, it is unlikely that new relationships would be created in a crisis situation where uncertainty about the robustness of other financial institutions prevails, therefore the fact that no new links are created is also reasonable.

B Illustration of shock transmission on the asset side

Let us illustrate the shock transmission on the asset side with a numerical example for $n = 3$. Initial total assets $\mathbf{q^0}$ of each bank are 10 units. Initial external assets are given by the column vector $\mathbf{l^0} = [4; 5; 6]$ and external assets after the shock are given by $\mathbf{l^1} = [1; 2.5; 2]$. The matrix of interbank exposures is

$$\mathbf{X} = \begin{pmatrix} 0 & 3 & 3 \\ 2 & 0 & 3 \\ 2.5 & 1.5 & 0 \end{pmatrix}, \tag{32}$$

which implies that total assets after the shock are $\mathbf{q^1} = \mathbf{Bl^1} = [3.3; 4.3; 3.5]$.

Figure 6: Impact on total assets of a shock to external assets for each bank traced for 9 rounds, where each round corresponds to a term in Equation 7 and 0 is the initial period

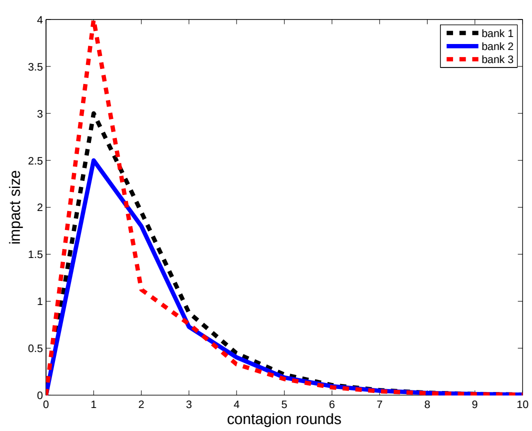


Figure 7: Cumulative impact on total assets of a shock to external assets for each bank traced for 9 rounds, where each round corresponds to a term in Equation 7 and 0 is the initial period

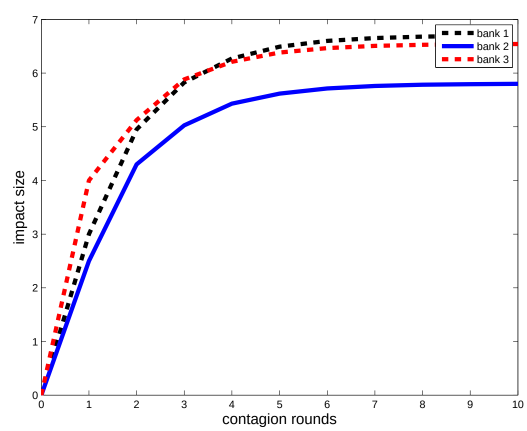


Figure 6 illustrates the decomposition of the change in total assets $\mathbf{\Delta q}$ defined in Equation 7, where 0 is the initial period and the impact of the shock on total assets of each bank is then traced for 10 rounds. Round 1 corresponds to the impact of the shock, round 2 is the direct effect of the shock and subsequent rounds capture the feedback effects. Figure 6 shows that the individual impact of each additional round gets smaller and smaller, eventually getting close to zero. Figure 7 illustrates the same decomposition, but now the effects are cumulated. We see that even though the effects tend to get smaller in each round, there is an amplification of the initial shock, since the cumulative effect increases after round 1. Amplification eventually levels offs, since the impact of higher rounds gets smaller and smaller.

C Fire sales without distinguishing trading and banking books

In this section we integrate price dynamics via fire sales into the framework, without making a distinction between banking and trading books, as in Greenwood et al. (2015). As in the main body of the paper, the analysis assumes there are three periods. Initial external assets in period 0 are given by $\mathbf{l^0} = \mathbf{W}\hat{\mathbf{h}}^{\mathbf{0}}\mathbf{p^0}$. In period 1 a shock to asset values occurs which leads to an updated vector $\mathbf{p^1}$ of relative prices between market and book values. The shock also triggers fire sales between periods 1 and 2.

The fire sales happen between periods 1 and 2. The detailed computational steps to calculate the fire sales are the following. In Step 1, an initial exogenous shock occurs. The shock is defined as a shock to asset returns $\mathbf{r^1} = (\mathbf{p^1} - \mathbf{p^0})$ which leads to a change in bank returns $\mathbf{\Pi^1} = \mathbf{W}\mathbf{r^1}$ in Step 2.

Banks sell assets to return to leverage targets after the shock occurs. Let $\mathbf{\Lambda}$ be the $n \times n$ diagonal matrix of leverage ratios of debt to equity, where each diagonal element is $\Lambda_{ii} = \frac{q_i^0 - e_i}{e_i}$. This matrix captures the pre-shock leverage positions of all banks, and is used within our framework as the target leverage banks aim for. In Step 3 the shortfall to regain target leverage is computed as:

$$\hat{\mathbf{l}}^{\mathbf{0}}\mathbf{\Lambda}\mathbf{Z}\mathbf{r^1} \text{ for the system } (n \times 1)$$
$$\hat{l}^0_{ii}\Lambda_{ii}\sum_k Z_{ik}r^1_k \text{ for bank } i$$

where $\mathbf{Z}$ is a weighting matrix. Each bank i holds a portfolio of m assets. z_{ik} is the ratio of asset k held by bank i over the total holdings of bank i.

Target exposures remain fixed in percentage terms, implying that banks sell assets proportionately to their existing holdings. In Step 4 net asset sales are then given by:

$$\mathbf{Z}'\hat{\mathbf{l}}^{\mathbf{0}}\mathbf{\Lambda}\mathbf{Z}\mathbf{r^1} \text{ for the system } (m \times 1)$$
$$\sum_i Z_{ik'}\hat{l}^0_{ii}\Lambda_{ii}\sum_k Z_{ik}r^1_k \text{ for asset } k'$$

where k' denotes one specific asset out of the m assets in vector $\mathbf{h}$. Let $\mathbf{\Phi}$ be the $m \times 1$ vector of net asset purchases by all banks during period 1. If banks keep their portfolio shares constant, then bank's net asset purchases are equal to $\mathbf{\Phi} = \mathbf{Z}'\hat{\mathbf{l}}^{\mathbf{0}}\mathbf{\Lambda}\mathbf{Z}\mathbf{r}^{\mathbf{1}}$ (as computed in Step 4 above). Period 2 external assets $\mathbf{h}^{\mathbf{2}}$ are then equal to the initial assets plus their net asset purchases $\mathbf{h}^{\mathbf{0}} + \mathbf{\Phi}$.

In Step 5, the final computational step, the price impact of the fire sale is computed according to a linear model where $\hat{\mathbf{\Theta}}$ is an $m \times m$ diagonal matrix with price impact coefficients for each asset:

$$\begin{array}{c} \hat{\mathbf{\Theta}}\mathbf{Z}'\hat{\mathbf{l}}^{\mathbf{0}}\mathbf{\Lambda}\mathbf{W}\mathbf{r}^{\mathbf{1}} \text{ for the system}(m \times 1) \\ \hat{\Theta}_{k'k'} \sum_i Z_{ik'} \hat{l}^0_{ii} \Lambda_{ii} \sum_k Z_{ik} r^1_k \text{ for asset } k' \end{array}$$

The price impact is related to the price in period 2 in the following way: $\mathbf{p}^{\mathbf{2}} - \mathbf{p}^{\mathbf{1}} = \hat{\mathbf{\Theta}}\mathbf{\Phi}$. The new price vector is then given by $\mathbf{p}^{\mathbf{2}} = \hat{\mathbf{\Theta}}\mathbf{\Phi} + \mathbf{p}^{\mathbf{1}}$.

Combining the above with Equation 11, we find

$$\begin{aligned} \mathbf{l}^{\mathbf{2}} &= \mathbf{Z}\hat{\mathbf{h}}^{\mathbf{2}}\mathbf{p}^{\mathbf{2}} \\ &= \mathbf{Z}\hat{\mathbf{h}}^{\mathbf{2}}(\hat{\mathbf{\Theta}}\mathbf{\Phi} + \mathbf{p}^{\mathbf{1}}) \end{aligned} \tag{33}$$

Equation 33 maps the shock to the individual assets into the new external assets of the individual banks. In order to include an interaction with the direct interbank exposures, we combine Equations 3 and 33 to obtain

$$\mathbf{q}^{\mathbf{2}} = \mathbf{B}\mathbf{l}^{\mathbf{2}} = \mathbf{B}\mathbf{W}(\hat{\mathbf{h}}^{\mathbf{0}} + \hat{\mathbf{\Phi}})(\hat{\mathbf{\Theta}}\mathbf{Z}'\hat{\mathbf{l}}^{\mathbf{0}}\mathbf{\Lambda}\mathbf{Z}\mathbf{r}^{\mathbf{1}} + \mathbf{p}^{\mathbf{1}}), \tag{34}$$

which provides one with the ability to analyse the interaction and amplification of direct and common exposures across banks. It is then possible to trace the impact of an initial shock over several contagion rounds as given by

$$\mathbf{q}^{\mathbf{2}} = \mathbf{l}^{\mathbf{2}} + \mathbf{A}\mathbf{l}^{\mathbf{2}} + \mathbf{A}\mathbf{A}\mathbf{l}^{\mathbf{2}} + \dots. \tag{35}$$

Finally, it is also possible to decompose the shock transmission in terms of changes:

$$\mathbf{\Delta q} = \mathbf{\Delta l} + \mathbf{A\Delta l} + \mathbf{AA\Delta l} + \dots, \tag{36}$$

where $\mathbf{\Delta q} = \mathbf{q}^{\mathbf{2}} - \mathbf{q}^{\mathbf{0}}$ and $\mathbf{\Delta l} = \mathbf{l}^{\mathbf{2}} - \mathbf{l}^{\mathbf{0}}$. Equation 36 can then be compared to an analogous decomposition without price dynamics as presented in Equation 13.

C.1 Vulnerability measures

Greenwood et al. (2015) define several measures to quantify vulnerability of the banking sector to fire sales. They define an aggregate vulnerability measure AV as the percentage of aggregate bank equity that would be wiped out if banks fire sell in response to a shock to asset values, *excluding* the initial impact of the shock.

$$AV = \frac{\mathbf{i}'\hat{\mathbf{l}}^{\mathbf{0}}\mathbf{Z\Theta\Phi}}{\mathbf{i}'\mathbf{e^0}}, \tag{37}$$

where $\mathbf{r^2} = \mathbf{p^2} - \mathbf{p^1}$.

The contribution that each bank has, through fire sales spillovers, on the aggregate vulnerability of the banking system is called the systemicness S_j of bank j. It is important to note that in order to compute the systemicness S_j of bank j, it is assumed that the shock only affects bank j. Therefore one needs to filter the impact of the shock by pre-multiplying it by $i_j i_j'$ when computing S_j, such that

$$S_j = \frac{\mathbf{i}'\hat{\mathbf{l}}^{\mathbf{0}}\mathbf{Z\Theta Z}'\hat{\mathbf{l}}^{\mathbf{0}}\mathbf{\Lambda} i_j i_j' \mathbf{Zr^1}}{\mathbf{i}'\mathbf{e^0}}. \tag{38}$$

Since systemicness S_j is the contribution of each bank through fire sales spillovers and/or scaling down of interbank exposures to the aggregate vulnerability of the banking system, $AV = \sum_{j=1}^{n} S_j$.

Indirect vulnerability IV_j measures the fraction of equity of bank j that is wiped out following the deleveraging at other banks and is given by

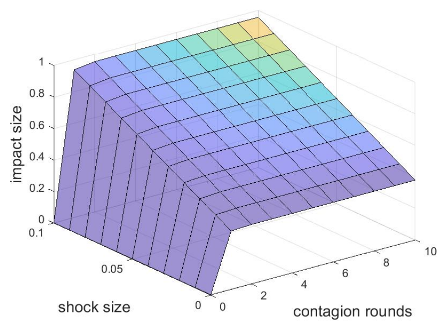
$$IV_j = \frac{i_j'\hat{\mathbf{l}}^{\mathbf{0}}\mathbf{Z\Theta Z}'\hat{\mathbf{l}}^{\mathbf{0}}\mathbf{\Lambda Zr^1}}{e_j^0}. \tag{39}$$

Direct vulnerability DV_j in turn measures the fraction of equity of bank j that is wiped out following the direct effect of the shock:

$$DV_j = \frac{\mathbf{i_j}'\hat{\mathbf{l}}^{\mathbf{0}}\mathbf{r^1}}{e_j^0}. \tag{40}$$

D Additional Figures

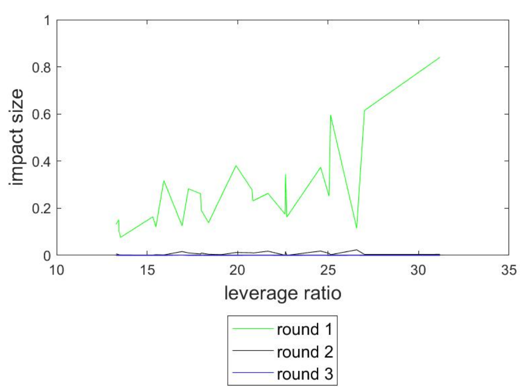
Figure 8: Cumulative impact by shock size



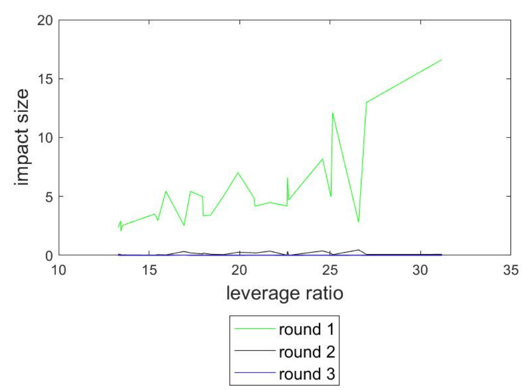
Notes: Ratio to total banking sector equity, fire sales case. Contagion rounds correspond to terms in Equation 20.

Figure 9: Impact as a function of leverage for first 3 contagion rounds

(a) No fire sales

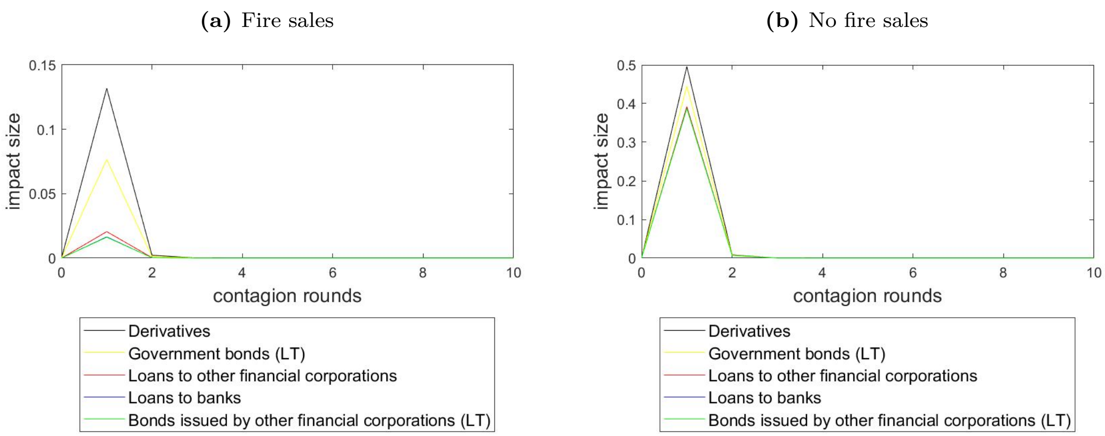


(b) Fire sales



Notes: Impact measured as losses as ratio to total banking sector equity. Rounds 1 to 3 corresponds to the three first terms in Equation 13 for the basic case and to the three first terms in Equation 20 for the fire sales case.

Figure 10: Impact by asset class



Notes: Impact measured as losses as ratio to total banking sector equity. Each of the rounds 1 to 10 corresponds to a term in Equation 13 for the basic case and to a term in Equation 20 for the fire sales case.

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