BIS Working Papers
No 1246

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by Paul Beaudry, Paolo Cavallino and Tim Willems

Monetary and Economic Department

March 2025 (revised July 2025)

JEL classification: E21, E43, E44, E52, G51

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This publication is available on the BIS website (www.bis.org).

ISSN 1020-0959 (print)
ISSN 1682-7678 (online)

Monetary policy along the yield curve: Why can central banks affect long-term real rates?*

Paul Beaudry, Paolo Cavallino, and Tim Willems†

This version: July 2025

Abstract

Evidence suggests that monetary policy can affect long-term real interest rates, but it is not clear what drives this. We argue this occurs because very persistent policy-induced interest rate changes have only weak effects on activity. This can arise when consumption-savings decisions are not primarily driven by intertemporal substitution, but also by life-cycle forces associated with retirement. Within such an environment, we show that the impact of highly persistent monetary policy shocks is determined by two forces: an asset valuation effect, and the response of the average marginal propensity to consume out of financial wealth. Our quantitative analysis indicates that these forces roughly cancel out, allowing monetary policy to (unconsciously) drive trends in long-run real rates. Our findings also imply that very precise knowledge of r* might not be essential to the successful conduct of monetary policy.

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*This is a revised version of a paper first circulated under the title "Life-cycle forces make monetary policy transmission wealth-centric" (NBER WP No. 32511, May 2024). We thank Michał Brzoza-Brzezina, Heejeong Kim, Tatiana Kirsanova, Sagiri Kitao, Christos Koulovatianos, Juanpa Nicolini, Monika Piazzesi, Min Wei, and Christian Wolf for their insightful conference discussions. Further thanks to Fernando Boffi, Ricardo Caballero, Mick Devereux, Wouter den Haan, Mike Joyce, Thijs Knaap, Étienne Latulippe, Jean-Paul L'Huillier, Ben Moll, Alessandro Rebucci, and various audiences for useful comments. The views expressed in this paper are those of the authors, and not necessarily those of the BIS, the Bank of England or its committees.

†Beaudry: University of British Columbia and NBER (Paul.Beaudry@ubc.ca); Cavallino: Bank of International Settlements (Paolo.Cavallino@bis.org); Willems: Bank of England and Centre for Macroeconomics (Tim.Willems@bankofengland.co.uk).

1 Introduction

Changes in long-term real rates continue to receive considerable attention. This includes understanding the secular decline in the decades prior to Covid, as well as recent changes in the opposite direction. The main class of explanations for these movements are *real* in nature, such as productivity growth, demographics, income inequality, and changes in the demand and supply of safe assets. One factor that is often dismissed is monetary policy – driven by the view that most long-term real economic outcomes are invariant to monetary policy beyond horizons long enough to allow prices to be reset.

From this perspective, it is puzzling that long-term real rates appear rather sensitive to changes in a central bank's policy rate. Cochrane and Piazzesi (2002), Piazzesi (2005), Hanson and Stein (2015), and Nakamura and Steinsson (2018) provide evidence of such sensitivity in U.S. data, while Hansen et al. (2019) do so for the U.K.; earlier evidence by Skinner and Zettelmeyer (1995) reported similar findings for not only the U.S. and U.K., but also Germany and France.[1]

An even more puzzling observation in light of the standard view, is the finding that nearly all of the post-1980 decline in long-term U.S. rates is driven by movements occurring in a narrow 3-day window around FOMC meetings (Hillenbrand, 2023). One interpretation is that central banks have superior information on the real determinants of long-term rates and that its decisions convey this information. This explanation has the appealing property of being consistent with the standard view that long-term real rates are driven by real forces. But it has the less attractive property of relying on central banks having substantial private information (or rare expertise) not directly available to markets. This, despite the latter having access to much of the same models and data, whilst also being populated by many former central bank employees.

An alternative, more direct interpretation is that central banks may drive real rates over long stretches of time. But that begs the question as to why very persistent rate changes would not have large effects on activity and inflation.

From the perspective of New Keynesian models, central banks are thought not to be able to affect long-term real rates due to their strong impact on activity. In these models, the potency of monetary policy shocks is increasing in their persistence. Accordingly, if a central bank tried to keep real rates away from their "natural" flexible-price level (r^*) for long, this would have strong effects on activity and inflation. Recognizing this, central

[1] Cochrane and Piazzesi (2002, p.91) nicely summarize the standard view: "Target changes seem to be accompanied by large changes in long-term interest rates (...) Can the Fed really raise the short rate 1 percent for five years or more, without leading to 1 percent lower inflation that would cancel any effect on longer yields?".

banks would want to avoid such outcomes, or correct course when noticing their long-term stance is away from r^*. As a result, they become *de facto* constrained to keeping their long-run policy stance consistent with the real forces shaping r^*.

But what if more persistent rate changes are *less* potent than temporary ones? Could reduced powers to affect activity in the long run imply greater control over long-term interest rates? Most importantly, are there reasons to question the notion that more persistent rate changes are more potent? This paper aims to shed light on these issues.

To fix ideas, let us express the link between excess demand and interest rates as:

$$\hat{y}_t = \mathbb{E}_t \sum_{j=0}^{\infty} \psi_j (r_{t+1+j} - r^*),$$

where $\hat{y}_t$ represents deviations in output from its natural level and $\mathbb{E}_t(r_{t+1+j} - r^*)$ captures expected deviations in real interest rates from r^*. Such a representation is consistent with – but more general than – a standard log-linearized New Keynesian model. Now suppose monetary policy is conducted so that the expected real rate deviates from r^* in a persistent fashion via $(r_t - r^*) = \rho(r_{t-1} - r^*) + \epsilon_t$. Then, the impact effect "$\Psi(\rho)$" on excess demand of a unit interest rate shock equals the persistence-weighted sum of horizon j-specific effects ψ_j, i.e., $\Psi(\rho) = \sum_{j=0}^{\infty} \psi_j \rho^j$. While the literature offers many estimates of $\Psi(\rho)$ for low values of ρ, knowing how $\Psi(\rho)$ behaves as ρ approaches 1 is what is relevant for understanding the effect of persistent deviations of r from r^*. If $\Psi(1)$ is large, persistent deviations create substantial excess demand. An inflation-targeting central bank would then need to ensure that r converges quickly to r^*. In this sense, r^* poses a constraint. However, if $\Psi(1)$ is close to zero, the central bank would have greater ability to keep r persistently away from r^* since it would not substantially affect activity and inflation.[2]

In most infinitely-lived agent models, the potency of monetary policy – as governed by $\Psi(\rho)$ – rises with the shock's persistence ρ due to the compounded power of intertemporal substitution.[3] But when thinking about the impact of very persistent rate changes, forces other than intertemporal substitution are likely important. Persistent rate changes affect working households' desire to accumulate wealth, whilst also changing consumption possibilities of retirees. These life-cycle forces are generally absent from New Keynesian

[2]Very persistent deviations of interest rates from r^* are most easily conceptualized as changes in the intercept of a Taylor rule (to the extent that they are not reflecting changes in the true r^*).

[3]For the baseline New Keynesian model, $\Psi(1) = -\infty$. This has raised issues like the Forward Guidance puzzle and initiated approaches that lead to a discounted Euler equation (Del Negro et al., 2013; McKay et al., 2016; Gabaix, 2020). However, even with a discounted Euler equation, the potency of a monetary shock always strengthens with its persistence, i.e., $\Psi'(\rho) < 0$, meaning that $\Psi(1)$ is negative and still quite sizeable.

models because they are predominantly used for short-term analyses, where ρ is assumed low. But since $\Psi(1)$ determines what happens if real rates were to persistently deviate from r^*, it is important to incorporate these lower frequency forces if ones wants to explore why and when monetary policy may be able to affect long-term rates.

To understand the effects of having monetary policy cause persistent deviations in r from r^*– that is for understanding the forces behind $\Psi(\rho)$ when ρ is close to 1 – this paper develops a Finitely-Lived Agent New Keynesian (FLANK) model. We show that such a model yields a rich but concise description of the relation between the path of future interest rates and activity.

A key insight is that the impact of highly persistent monetary policy shocks can be reduced to two simple effects. First, there is a valuation effect for assets with positive duration, working in the conventional direction (with higher rates lowering demand). Second, there is an effect on the marginal propensity to consume (MPC) out of financial wealth. This effect tends to work in the *unconventional* direction – leaving a net total effect which implies that persistent rate changes might not affect excess demand much (or even with the unconventional sign).

To understand why, consider a retired household, or one saving to retire in the future. It is not clear they should increase consumption in response to capital gains resulting from persistently lower rates (Auclert, 2019; Moll, 2020; Fagereng et al., 2021; Greenwald et al., 2023). The reason is that the typical household is "short duration" by having a prospective labor income stream that is of shorter duration than their prospective consumption stream (due to the presence of a retirement phase). As a result, when rates fall, households may see the present discounted value of their liabilities go up by more than that of their assets – making them want to hold more units of assets, to compensate for each unit now yielding less. The existence of such an "interest income effect" implies that the aggregate MPC out of financial wealth may well *decrease* when rates fall in a persistent fashion.[4] This works in the unconventional direction, with lower rates *dampening* demand. Since the asset valuation effect operates in the conventional direction, the competing forces may

[4]This is consistent with Ring (2024), who empirically finds that wealth taxation – which lowers the rate of return – *increases* savings; there are studies reporting the opposite (Jakobsen et al., 2020) but, as argued in Brülhart et al. (2022) and Ring (2024), those findings may in part be driven by tax evasion/avoidance – as opposed to the pure consumption-savings response. The notion that income effects may dominate intertemporal substitution is also supported by the observation that retirees do not dissave much (De Nardi et al., 2016; Fella et al., 2024; Auclert et al., 2024), which mainly leaves the return on savings for consumption (also see Daniel et al. (2021) and Crawley (2025), who note this behavior is in line with popular investment advice). Rajan (2013) already worried that the post-GFC era of persistently low rates might not be expansionary because "savers put more money aside as rates fall in order to meet the savings they think they will need when they retire". Studies like Nabar (2011), Aizenman et al. (2019), Van den End et al. (2020), and Ahmed et al. (2024) find supporting evidence in aggregate data.

largely offset each other – which is what we find for reasonable calibrations.

In the knife-edge case of perfect offset ($\Psi(1) = 0$) a central bank would no longer be (locally) constrained by an r^*. Monetary policy, even if not aimed to do so, would then become an important driver of long-run real rates. In the more plausible case we advance, where the sum of the two forces is small but not exactly 0, precise knowledge of r^* is still not very relevant as interest rates can be kept away from r^* "for long" (affecting long-term rates via standard term structure theory) without major effects on excess demand and inflation.[5] Knowing the exact location of r^* then becomes of diminished relevance for monetary policy purposes, as r^* does not put a tight constraint on the long-term real rates that a central bank can implement. One could say that r^* becomes quasi-irrelevant in this case, as the system becomes very "forgiving" towards a central bank working with a wrong view of r^*.[6] Instead, it is the central bank's perception of r^* that can emerge as an important driver of long-term real rates.

The above intuition is illustrative. Our model enables us to analyze under what conditions such mechanisms emerge in general equilibrium. This will depend on several factors, including the expected duration of working and retirement phases, and average asset duration. But a key parameter is the elasticity of intertemporal substitution (EIS). For $EIS \geq 1$, FLANK behaves much like standard infinitely-lived agent models, with the potency of monetary policy always increasing in persistence. Central banks then cannot affect long-term real rates without creating strong inflation or deflation. In contrast, for $EIS < 1$ (a case with strong empirical support[7]) the MPC out of wealth can become *increasing* in the real rate of interest, thus countering valuation effects. Very persistent rate changes may then have only small effects on activity. If the Phillips curve is locally quite flat, persistent rate changes might also have minimal impact on inflation.

Motivating evidence. The life-cycle forces we focus on suggest that households' desire to save is significantly shaped by the interest income they expect to receive on their assets. When rates are lowered, this tends to increase wealth holdings through valuation effects. However, it is not clear that this should boost consumption, as *desired* wealth holdings may rise simultaneously (to compensate for the lower interest income per unit held). Without controlling for interest rate effects on asset demand, the link between

[5] This choice could nonetheless have important implications for asset valuations and wealth inequality, the analysis of which we leave for future work.

[6] It is important to stress that our analysis is done within the confines of a closed economy (see Cesa-Bianchi et al. (2023), Obstfeld (2023), and Auclert et al. (2024) for open economy considerations). In this regard, our analysis is best thought of as applying to a rather large economy (like the U.S.).

[7] See for example Yogo (2004) and Best et al. (2020) and Ring (2024), who all estimate the $EIS << 1$.

consumption and wealth may thus be weak. In contrast, when controlling for interest rates, consumption and "rate-adjusted wealth" should comove positively – as people will want to spend wealth holdings in excess of desired levels.

The potential relevance of this logic can be seen in Figure 1. Panel (a) plots the natural log of detrended U.S. real consumption per capita ($\ln C_t$) against the natural log of detrended beginning-of-period real U.S. wealth holdings per capita ($\ln W_{t-1}$) over 1982Q1-2019Q4.[8] There is very little relationship between the two, with their correlation amounting to an insignificant 0.056. At face value, this may suggest that there is no link between fluctuations in wealth and consumption.

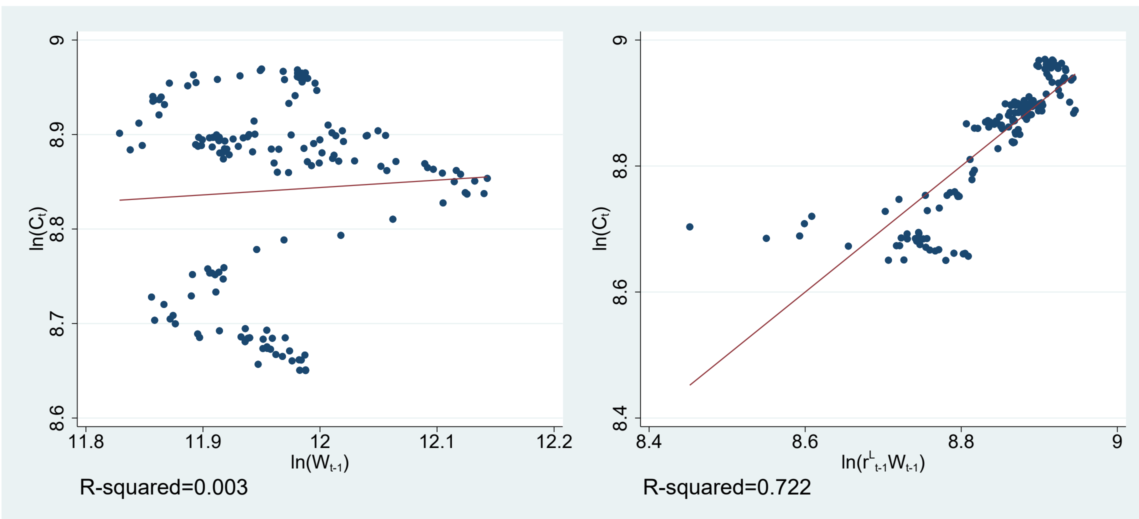

Figure 1: Scatter plot illustrating the correlation between detrended U.S. real consumption levels and detrended real wealth holdings. Panel (a) features no adjustment for the level of interest rates; Panel (b) looks at the product $r^{LT}W$. Quarterly data from 1982Q1-2019Q4.

This link may be weak because "raw" wealth does not accurately capture consumption possibilities, as it neglects the flow-aspect (the holder of $500,000 can afford to consume more when those assets yield 5%, as opposed to only 0.5%). Under this logic, consumption should be driven by something closer to *the product of* the interest rate and wealth holdings, as that captures both dimensions (stock and flow). Panel (b) of Figure 1 plots the same variables as Panel (a), except that wealth is now multiplied by a long-term real

[8] Data are quarterly and available from FRED starting 1982Q1. Consumption has code PCE; wealth has code TABSHNO. Price deflation is done using the CPI (CPIAUCSL); per-capita amounts are obtained through division by POPTHM. Consumption and wealth are made stationary by linear detrending using the pre-GFC average growth rate of real GDP per capita (0.54% per quarter). Over the entire period, real GDP grew at a quarterly rate of 0.4%. Using this detrending factor gives similar results, but we detrend using the higher pre-GFC growth rate as our paper suggests that the post-GFC period may have had low growth because of monetary policy's inability to push the economy towards its potential.

rate, i.e., we are now looking at the correlation between $\ln C_t$ and $\ln(r_t^{LT} W_{t-1})$.[9] This simple adjustment has a striking effect on the correlation: it jumps to 0.850 and is very significant. The data thus suggest that consumption is much more closely related to a "wealth-flow" concept, than to "raw" wealth.

In the remainder of this paper we extend a standard New Keynesian model with life-cycle forces – showing how this modifies the link between consumption, interest rates, and wealth holdings in a way consistent with Figure 1.

Outline. After discussing the related literature in Section 2, Section 3 introduces our FLANK model. Section 4 covers the model's implications for monetary policy, explaining how we can simultaneously have short-lived rate cuts being expansionary while persistent cuts have little effect. This section clarifies the forces determining $\Psi(\rho)$, and especially $\Psi(1)$. Section 5 shines further light on why precise knowledge of r^* may be considered quasi-irrelevant for monetary policymaking. Section 6 discusses some of the model's assumptions and relevant extensions, after which Section 7 concludes.

2 Related literature

Our paper relates to several earlier works. Gertler's (1999) OLG structure, which we deploy, has been used to analyze issues related to monetary policy by, among others, Sterk and Tenreyro (2016) and Galí (2021). Sterk and Tenreyro focus on a redistribution channel of monetary policy when prices are fully flexible, while Galí's work analyzes the conduct of monetary policy amidst bubble-driven fluctuations. Fujiwara and Teranishi (2008) examine the impact of demographics on r^*, whilst also studying the distributional impact monetary policy may have on workers versus retirees. Bielecki et al. (2022) offer a more general OLG model to analyze the heterogeneous impact monetary policy can have across generations; Eggertsson et al. (2019) use an OLG model to formalize thinking about "secular stagnation". Our paper, in contrast, focuses on the impact that a retirement savings motive has on the monetary transmission mechanism and the resulting powers of central banks over long-run outcomes and interest rates.[10]

[9] This real rate is taken as the ex-ante 10-year real rate, available from FRED via code REAINTRATREARAT10Y.

[10] Del Negro et al. (2013) introduce finite lives into a New Keynesian setup via the Blanchard-Yaari route, thus abstracting from retirement. To mitigate their Forward Guidance puzzle, they need very high levels of mortality risk – implying an expected lifetime of about 11 years (although, as they point out, OLG models can be seen as proxies for models of agents hitting liquidity constraints). Thanks to the explicit modeling of retirees (whose behavior is quite different to that of workers) our model can

Second, our work relates to papers questioning whether lower interest rates are always expansionary. Bilbiie (2008) features "inverted aggregate demand logic" stemming from limited asset market participation. In Mian et al. (2021) monetary stimulus promotes debt accumulation, which – while being stimulative in the short run – ultimately starts forming a drag on the economy, as savers have lower MPCs in their model. Abadi et al. (2023), Eggertsson et al. (forthcoming), and Cavallino and Sandri (2023) also present frameworks in which rate cuts can be contractionary, due to an adverse impact on the banking sector or capital flows. In contrast, our model emphasizes that the link between activity and interest rates may vary along the yield curve. There is also the "neo-Fisherian" literature which explores the possibility that a persistent increase in rates might help to raise inflation (Schmitt-Grohé and Uribe, 2014; Cochrane, 2018).

Our model also links to the literature investigating the ability of monetary policy to affect long-term real rates. Nakamura and Steinsson (2018), Hansen, McMahon and Tong (2019), and Hillenbrand (2023) explain this via a central bank information effect, while Rungcharoenkitkul and Winkler (2023) allow for two-sided learning (with markets not just learning from the central bank, but the reverse occurring as well). Hanson and Stein (2005) allude to the impact of monetary policy on term premia, Bianchi et al. (2022) and Pflueger and Rinaldi (2022) focus on the impact on the equity premium, while Beaudry et al. (2024) develop a model featuring r^*-multiplicity (with monetary policy affecting which equilibrium gets to prevail). We do not wish to deny that these factors play a role, but propose a novel mechanism that has different implications – based on the "quasi-irrelevance of r^*" in a New Keynesian-style model where the true r^* continues to be pinned down in a unique way. Our explanation aligns well with the empirical results in Rigon (2022) and Hofmann et al. (2025), who report that Hillenbrand's (2023) finding mostly runs through changes in expected (real) short rates – not through information effects or term premia.

Finally, we build on papers that have enriched the New Keynesian model with additional transmission mechanisms relating to asset prices (Caballero and Simsek, 2024; Caballero et al., 2025) and agent heterogeneity, such as the "TANK/HANK" literature (Galí et al., 2007; Bilbiie, 2008; Oh and Reis, 2012; Gornemann et al., 2014; Werning, 2015; Guerrieri and Lorenzoni, 2017; Ravn and Sterk, 2017; Den Haan et al., 2018; Kaplan et al., 2018; Debortoli and Galí, 2024; Auclert et al., 2025).[11] Our work also relates

be calibrated in a way that is consistent with the data on expected working/retired lives and still solve various puzzles.

[11] In light of the many TANK/HANK papers, which are about allowing for idiosyncratic income risk, consideration of retirement risk seems a natural complementary route as this can be seen as the (very high) risk of becoming "unemployed" in a lasting way towards the end of one's life, potentially due to an

to Auclert (2019) who analyzes the impact of transitory rate changes – showing how unhedged interest rate exposure, distinguishing solely between net assets that pay "today" versus "in the future", is sufficient for the first-order response of consumption to shocks. For persistent rate changes, the exact timing of cash flows starts to matter. In light of this, Greenwald et al. (2023) develop a life-cycle model to understand how the observed, persistent decline in real rates has affected wealth inequality, also documenting how lower rates contract consumption possibilities for "the young" who have not yet accumulated many financial assets with positive duration, but have a long consumption stream to finance going forward.

3 A life-cycle model for monetary policy

This section describes our model.[12] As we adopt a common production setup – monopolistically competitive firms facing price adjustment frictions – and combine this with life-cycle consumption-savings decisions, one can refer to this model as "FLANK", for Finitely-Lived Agent New Keynesian model. We model all households as optimizers. This may not seem realistic given the evidence on the presence of "hand-to-mouth" households. But, as we discuss in Section 6, we do not think this modelling choice hinders the model's main insights (even if we agree that optimizers represent only a fraction of the population).

Environment. There is a measure one of households, subject to a life cycle as in Gertler (1999, which – in turn – built on Yaari (1965) and Blanchard (1985)). Each household starts life in a working state and transits out with Poisson probability δ_1 – either due to being sent to retire, or because of a health shock preventing further work. At this transition, the household enters retirement where it faces Poisson death probability δ_2. Deceased households are immediately replaced by new, working households, implying that the share of workers is constant at $\vartheta = \frac{\delta_2}{\delta_1+\delta_2}$.

Retired Households. A retired household derives income from its financial wealth, reflecting past savings and a possible lump-sum public pension payment. Retirees invest their wealth in a portfolio of short- and long-term bonds. Short-term bonds are one-period assets whose gross nominal return, i_t, is set by the central bank. Their real return is $r_{t+1} \equiv i_t/\pi_{t+1}$, where π denotes the gross inflation rate. We model long-term bonds as

adverse health shock. Borella et al. (2025) report that around 80% of U.S. wealth holdings are driven by retirement, health care, and bequest motives; wage risk is found to account for around 10%.

[12] The real side of the model shares many features with the continuous time model in Beaudry et al. (2024). Our model departs from Beaudry et al. (2024) in that it is set in discrete time, is stochastic, allows for long-term debt, and is embedded in a New Keynesian setup.

real perpetuities with coupons that decay geometrically at rate μ (Woodford, 2001). A bond issued in period t then pays $(1-\mu)^h$ units of consumption $h+1$ periods later; hence, the bond's duration decreases in μ ($\mu = 1$ reduces this bond to a one-period instrument). The gross return on the long-term bond is:

$$r_{t+1}^b = \frac{1 + (1-\mu)\, q_{t+1}}{q_t},$$

where q_t is the long-term bond's price. The optimization problem faced by a retired household j with CRRA-preferences (where σ is the coefficient of relative risk aversion, making $1/\sigma$ the EIS) reads:

$$V_t^r\left(\tilde{a}_t^j\right) = \max_{c_t^j, \alpha_t^j, \tilde{a}_{t+1}^j} \left\{ \frac{\left(c_t^j\right)^{1-\sigma}}{1-\sigma} + (1-\delta_2)\, \beta_t \mathbb{E}_t \left[V_{t+1}^r\left(\tilde{a}_{t+1}^j\right)\right] \right\},$$

$$s.t.\ \tilde{a}_{t+1}^j = r_{t+1}^j \left(\tilde{a}_t^j - c_t^j\right), \tag{1}$$

$$r_{t+1}^j = r_{t+1} + \left(r_{t+1}^b - r_{t+1}\right) \alpha_t^j \tag{2}$$

where c_t^j is consumption, $\alpha_t^j \equiv \left(q_t b_t^j\right)/a_t^j$ is the share of wealth invested in long-term bonds "b", and $\tilde{a}_t^j \equiv r_t^j a_{t-1}^j$ is the beginning-of-period t stock of wealth held by household j, such that the real rate of return r_t^j works on whatever is left after period-$(t-1)$ consumption has been financed, i.e., on $a_{t-1}^j = \tilde{a}_{t-1}^j - c_{t-1}^j$. Finally, $\beta_t \equiv \beta e^{\varepsilon_t^\beta}$, where ε_t^β is a demand shifter. Optimal consumption satisfies:

$$\left(c_t^j\right)^{-\sigma} = (1-\delta_2)\, \beta_t \mathbb{E}_t \left[\frac{dV^r\left(\tilde{a}_{t+1}^j\right)}{d\tilde{a}_{t+1}^j} r_{t+1}^j \right], \tag{3}$$

with the portfolio optimality condition:

$$0 = \mathbb{E}_t \left[\frac{dV^r\left(\tilde{a}_{t+1}^j\right)}{d\tilde{a}_{t+1}^j} \left(r_{t+1}^b - r_{t+1}\right) \right]. \tag{4}$$

At the same time, the envelope theorem implies that:

$$\frac{dV_t^r\left(\tilde{a}_t^j\right)}{d\tilde{a}_t^j} = \left(c_t^j\right)^{-\sigma}, \tag{5}$$

so that (4) boils down to:

$$0 = \mathbb{E}_t \left[\left(c_{t+1}^j\right)^{-\sigma} \left(r_{t+1}^b - r_{t+1}\right) \right].$$

Guessing that $V_t^r\left(\tilde{a}_t^j\right) \equiv \frac{\left(\tilde{a}_t^j\right)^{1-\sigma}}{1-\sigma}\left(\Gamma_t^j\right)^{-\sigma}$, with Γ_t^j conjectured to be a function of the future path of r_t^j and independent of $\tilde{a}_t^j$, this gives:

$$\frac{dV_t^r\left(\tilde{a}_t^j\right)}{d\tilde{a}_t^j} = \left(\tilde{a}_t^j\Gamma_t^j\right)^{-\sigma}. \tag{6}$$

By combining (5) and (6) we obtain:

$$\left(c_t^j\right)^{-\sigma} = \left(\tilde{a}_t^j\Gamma_t^j\right)^{-\sigma} \Leftrightarrow c_t^j = \tilde{a}_t^j\Gamma_t^j, \tag{7}$$

which we can plug into (1) to yield:

$$\tilde{a}_{t+1}^j = r_{t+1}^j\tilde{a}_t^j\left(1-\Gamma_t^j\right). \tag{8}$$

Finally, plugging (6), (7), and (8) into (3) gives a non-linear difference equation for Γ_t:

$$\left[\left(\Gamma_t^j\right)^{-1}-1\right]^{\sigma} = \left(1-\delta_2\right)\beta_t\mathbb{E}_t\left[r_{t+1}\left(\Gamma_{t+1}^jr_{t+1}^j\right)^{-\sigma}\right]. \tag{9}$$

This verifies our guess that Γ_t^j is independent of $\tilde{a}_t^j$, confirming that it is only a function of expected future rates and demand shocks. It is useful to note from (7) that Γ_t^j equals the MPC out of (beginning of period) financial wealth for retirees, which plays an important role to the interpretation of our findings later on.

We can thus write the utility of retirees as $V^r\left(\tilde{a}_t^j,\Gamma_t^j\right) = (1-\sigma)^{-1}\left(\tilde{a}_t^j\right)^{1-\sigma}\left(\Gamma_t^j\right)^{-\sigma}$, where V^r depends both on the stock of assets with which the household enters retirement ($\tilde{a}_t^j$) as well as on the entire future path of interest rates working over that stock (captured via Γ_t). Given the value of assets $\tilde{a}_t^j$, retired households are better off when rates are expected to be high, as this offers them a superior stream of interest income.

Let $c_t^r \equiv \int_{\mathbf{R}_{r,t}} c_t^j dj/\left(1-\vartheta\right)$ be the consumption of the representative retiree and define $a_t^r \equiv \int_{\mathbf{R}_{r,t}} a_t^j dj/\left(1-\vartheta\right)$ as their (end of period) financial wealth, where $\mathbf{R}_{r,t}$ denotes the set of retired households at time t. Given that all retired households choose the same asset portfolio, that is $\alpha_t^j = \alpha_t^r$ for all $j \in \mathbf{R}_{r,t}$, this implies $\Gamma_t^j = \Gamma_t$ for all $j \in \mathbf{R}_{r,t}$. Therefore:

$$c_t^r = a_t^r\left[\left(\Gamma_t^j\right)^{-1}-1\right]^{-1},$$

where $\left[\left(\Gamma_t^j\right)^{-1}-1\right]^{-1}$ reflects the MPC out of (end of period) financial wealth of the representative retiree, with a_t^r evolving as:

$$a_{t+1}^r = \left[(1-\delta_2)\, a_t^r r_{t+1}^r + \delta_2 \left(a_t^w r_{t+1}^w + \tau_{t+1}^r\right)\right] \left(1-\Gamma_{t+1}^j\right).$$

where τ^r is the lump-sum transfer received by households upon retirement. It can be seen as a public pension transfer that is paid once to the household upon retiring, and thereafter managed by the household.

Working Households. Next, consider a working household. It receives a real wage w_t for any labor input ℓ_t it provides, plus transfers from good-producing firms and transfers from/to the government. Workers face a δ_1 probability of moving into retirement next period. Their decision problem reads:

$$V_t^w\left(\tilde{a}_t^j\right) = \max_{c_t^j,\ell_t^j,\alpha_t^j,\tilde{a}_{t+1}^j} \left\{ \frac{\left(c_t^j\right)^{1-\sigma}}{1-\sigma} - \frac{\left(\ell_t^j\right)^{1+\varphi}}{1+\varphi} + \beta_t \mathbb{E}_t \left[(1-\delta_1)\, V_{t+1}^w\left(\tilde{a}_{t+1}^j\right) + \delta_1 V_{t+1}^r\left(\tilde{a}_{t+1}^j + \tau_{t+1}^r\right)\right] \right\},$$

$$s.t.\ \tilde{a}_{t+1}^j = r_{t+1}^j \left(\tilde{a}_t^j - c_t^j + \ell_t^j w_t + z_t^j + \tau_t^w + \tau_t^n\right),$$

$$r_{t+1}^j = r_{t+1} + \left(r_{t+1}^b - r_{t+1}\right)\alpha_t^j$$

where z_t^j represents dividends received from good-producing firms. τ_t^w and τ_t^n both represent tax/transfer schemes. τ_t^w is a tax used by the government to pay expenditures and interest on debt. τ_t^n is tax or transfer scheme which ensures that the inheritance received by newly-born households allows them to resemble existing working households – implying that we can consider a representative working household. The optimality conditions give rise to the following Euler equation:

$$\left(c_t^j\right)^{-\sigma} = \beta_t \left\{(1-\delta_1)\, \mathbb{E}_t \left[\left(c_{t+1}^j\right)^{-\sigma} r_{t+1}\right] + \delta_1 \mathbb{E}_t \left[\left(\tilde{a}_{t+1}^j + \tau_{t+1}^n\right)^{-\sigma} \Gamma_{t+1}^{-\sigma} r_{t+1}\right]\right\}, \tag{10}$$

supplemented by the portfolio decision and the labor supply schedule:

$$0 = \mathbb{E}_t \left[\left\{(1-\delta_1)\left(c_{t+1}^j\right)^{-\sigma} + \delta_1 \frac{dV_{t+1}^r\left(\tilde{a}_{t+1}^j\right)}{d\tilde{a}_{t+1}^j}\right\}\left(r_{t+1}^b - r_{t+1}\right)\right],$$

$$w_t = \left(c_t^j\right)^{\sigma} \left(\ell_t^j\right)^{\varphi}.$$

Note how the Euler equation for working households (10) features two terms on the RHS: the first term is familiar from models without retirement and implies that a lower interest rate, *ceteris paribus*, decreases the household's desire to save; this is standard intertemporal substitution. The second term on the RHS of (10), however, stems from the introduction of the prospect of retirement and shows how consumption is driven by wealth ($\tilde{a}_{t+1}^{j}$) adjusted for the expected path of interest rates (as captured by $\Gamma_{t+1}^{-\sigma} r_{t+1}$).

Since the assets of new and existing working households are equalized via the transfer τ^{n}, workers can be treated as homogeneous. Let c_t^w denote the consumption of the representative worker and a_t^w its end-of-period financial wealth. Then, c_t^w solves:

$$(c_t^w)^{-\sigma} = \beta_t \left\{ (1-\delta_1)\, \mathbb{E}_t \left[\left(c_{t+1}^w \right)^{-\sigma} r_{t+1} \right] + \delta_1 \mathbb{E}_t \left[\left(a_t^w r_{t+1}^w \Gamma_{t+1} \right)^{-\sigma} r_{t+1} \right] \right\},$$

where a_t^w evolves as:

$$a_{t+1}^w = (1-\delta_1)\, a_t^w r_{t+1}^w + \delta_1 a_t^r r_{t+1}^r - c_{t+1}^w + \ell_{t+1} w_{t+1} + z_{t+1} + \tau_{t+1}^w.$$

Good-producing firms. Each working household $j \in \mathbf{R}_{w,t}$ owns a firm that produces a differentiated good using the technology $y_t^j = A \ell_t^j$. Upon retiring, households liquidate their firms which are replaced by new ones owned by new working households. Firms are monopolistically competitive and set prices subject to a quadratic adjustment cost (Rotemberg, 1982). Let P_t^j be the price chosen by firm j at time t and $\pi_t^j \equiv P_t^j / P_{t-1}^j$ be its growth rate. Then, the firm pays adjustment cost $\Theta\left(\pi_t^j\right) = y_t^j \frac{\theta}{2} \left(\pi_t^j - \bar{\pi}\right)^2$, where $\bar{\pi}$ is the inflation target and θ governs the cost of adjusting prices. The resulting Phillips curve takes the standard form (which, to a first-order approximation, has the same reduced form as under Calvo-pricing; Roberts (1995)):

$$(\pi_t - \bar{\pi})\, \pi_t = \lambda \left(\frac{\epsilon}{\epsilon - 1} m c_t - 1 \right) + \mathbb{E}_t \left[\Lambda_{t,t+1}^w \left(\pi_{t+1} - \bar{\pi} \right) \pi_{t+1} \frac{y_{t+1}}{y_t} \right],$$

where $\lambda \equiv (\epsilon - 1)/\theta$ represents the slope of the Phillips curve and ϵ is the elasticity of substitution between product varieties,[13] $y_t = \int_{\mathbf{R}_{w,t}} y_t^j dj$ denotes aggregate output, while $\Lambda_{t,t+1}^w$ is the stochastic discount factor of the representative working household:

$$\Lambda_{t,t+1}^w = \beta_t \frac{(1-\delta_1)\left(c_{t+1}^w\right)^{-\sigma} + \delta_1 \left(a_t^w r_{t+1}^w \Gamma_{t+1} \right)^{-\sigma}}{\left(c_t^w\right)^{-\sigma}}.$$

This captures the familiar notion that households place more weight on the future when their marginal utility is high, but it features the additional forces stemming from retirement: households now place more weight on the future when they hold fewer assets a_t^w or when the interest rate path is lower (as captured via Γ).

[13] Households consume a CES aggregate of all varieties: $c_t^j = \left[\int_{\mathbf{R}_w} c_t^j (j)^{\frac{\epsilon-1}{\epsilon}} dj \right]^{\frac{\epsilon}{\epsilon-1}}$.

The real marginal cost of production is $mc_t = (1-\tau_t)\, w_t/A$, where τ_t is a wage subsidy financed through lump-sum taxes levied directly on good-producing firms. We use this subsidy to undo the steady-state markup and to eliminate the impact of labor supply wealth effects on inflation, such that $mc_t = \frac{\epsilon-1}{\epsilon}\left(\frac{y_t}{\vartheta A}\right)^{1+\varphi}$. Since firms are identical, the real dividend generated by each firm is $z_t = \frac{y_t}{\vartheta}\left[1-\frac{\theta}{2}\left(\pi_t-\bar{\pi}\right)^2\right] - \ell_t w_t$.

Government. The government's budget constraint reads:

$$s_t^g + q_t b_t^g = q_{t-1} b_{t-1}^g r_t^b + s_{t-1}^g r_t + \vartheta \tau_t^w + \vartheta \delta_1 \tau_t^r,$$

where s_t^g and b_t^g are the supply of short- and long-term government bonds, respectively. Without loss of generality, we take the limit for $s_t^g \downarrow 0$ and assume $b_t^g = b^g$, for all $t \geq 0$. This implies that tax policy must satisfy $\vartheta \tau_t^w + \vartheta \delta_1 \tau_t^r = -b^g \left(1-\mu q_t\right).$

The central bank sets monetary policy according to the following rule:

$$i_t = r^* \bar{\pi} \left(\frac{\mathbb{E}_t\left[\pi_{t+1}\right]}{\bar{\pi}}\right)^{1+\phi} e^{\varepsilon_t^i}, \tag{11}$$

where $\phi > 0$ governs the central bank's responsiveness to expected inflation-deviations from target ($\bar{\pi}$), r^* is the steady-state real interest rate, and ε_t is a monetary policy shock.

Market clearing . Market clearing requires:

$$\begin{aligned}
\vartheta c_t^w + (1-\vartheta)\, c_t^r &= y_t \left[1 - \frac{\theta}{2}\left(\pi_t - \bar{\pi}\right)^2\right], \\
\vartheta a_t^w + (1-\vartheta)\, a_t^r &= q_t b^g, \\
\vartheta b_t^w + (1-\vartheta)\, b_t^r &= b^g,
\end{aligned}$$

where $b_t^r \equiv \int_{\mathbf{R}_{r,t}} b_t^j dj / \left(1-\vartheta\right)$ and $b_t^w \equiv \int_{\mathbf{R}_{w,t}} b_t^j dj/\vartheta$ are the long-term bond holdings of the representative retiree and the representative worker, respectively.

Exogenous Processes. We allow the model to be hit by two types of shocks: first, a standard monetary policy shock "ε_t^i" to the Taylor rule (11) and, second, a demand shock to β, ε_t^β. The exogenous variables ε_t^i and ε_t^β are assumed to follow AR(1) processes:

$$\varepsilon_t^i = \rho_i \varepsilon_{t-1}^i + \sigma_i \epsilon_t^i, \tag{12}$$

$$\varepsilon_t^\beta = \rho_\beta \varepsilon_{t-1}^\beta + \sigma_\beta \epsilon_t^\beta, \tag{13}$$

with the innovations "ϵ^i" and "ϵ^β" following a standard-normal distribution (σ_i and σ_β scale the shocks' standard deviations).

We furthermore assume a zero inflation target ($\bar{\pi} = 1$). The equilibrium and steady-state equations of our full model can be found in Appendix A.

4 Model properties: analytical and quantitative

To highlight how retirement preoccupations affect monetary policy, we simplify our model to derive analytical results that clarify the key mechanisms. Our simplifying assumptions lead to a compact system that can be handled almost as easily as the standard New Keynesian model, while simultaneously capturing forces stemming from life-cycle considerations. We then derive a "term structure representation" of the Euler equation, which shows how interest rates at different horizons affect activity differently. This enables us to discuss when and why our framework implies that the potency of monetary policy may be decreasing in the persistence with which it is conducted. This, we will argue, has important implications for how monetary policy may be able to affect long-term real rates without having much effect on inflation.

4.1 Simplifying the model

To provide a model which can be easily compared to a standard New Keynesian model, we assume that the transfer received by households upon retirement, τ^r, is designed to keep the distribution of financial wealth between workers and retirees constant at its steady-state level.[14] This enables us to obtain analytical solutions, while we shall later show that it is not driving the model's implications – neither qualitatively nor quantitatively. We set the level of government debt, b^g, so that the steady-state real interest rate (r^*) equals $1/\beta$. This ensures that the log-linearized system nests the standard representative agent New Keynesian ("RANK") model for $\delta_1 = 0$ (when every household remains in its working state *ad infinitum*). Finally, for the main propositions, we will focus on the case where $\delta_2 < \mu$, which implies that the expected duration of retirement is greater than the average duration offered by bonds.[15] This means that, in equilibrium, households' saving efforts in the asset with positive duration cannot fully close their negative duration gap (stemming from the need to finance consumption in retirement).[16]

[14]To simplify the algebra, the time-varying nature of the transfer is unexpected, so that workers do not anticipate receiving a transfer that varies with the state of the economy. This assumption is not necessary for our main results, but does make the presentation more transparent.

[15]Our propositions technically only require the weaker condition that $(1 - \delta_2)^{1/\sigma} > 1 - \mu$ (which is easily satisfied when σ is large), but imposing the stronger condition $\delta_2 < \mu$ eases exposition.

[16]This is clear to pension funds (to whom many have outsourced the process of saving for retirement): pension funds often have negative duration gaps of about 10 years, which forced many

With these simplifications, the log-linearized equilibrium reads:

$$\hat{y}_t = (1-\gamma)\,\hat{c}^w_t + \gamma\hat{c}^r_t \tag{14}$$

$$\hat{c}^r_t = \hat{q}_t + \left[\beta\,(1-\delta_2)^{\frac{1}{\sigma}}\right]^{-1}\hat{\Gamma}_t \tag{15}$$

$$\hat{c}^w_t = (1-\delta_1)\left(\mathbb{E}_t\hat{c}^w_{t+1} - \frac{1}{\sigma}\mathbb{E}_t\hat{r}_{t+1}\right) + \delta_1\left(\hat{q}_t + \left[\beta\,(1-\delta_2)^{\frac{1}{\sigma}}\right]^{-1}\hat{\Gamma}_t\right) - \frac{1-\delta_1}{\sigma}\varepsilon^\beta_t \tag{16}$$

$$\hat{\Gamma}_t = \beta\,(1-\delta_2)^{\frac{1}{\sigma}}\left[\mathbb{E}_t\hat{\Gamma}_{t+1} + \frac{\sigma-1}{\sigma}\mathbb{E}_t\hat{r}_{t+1} - \frac{1}{\sigma}\varepsilon^\beta_t\right] \tag{17}$$

$$\hat{q}_t = \beta\,(1-\mu)\,\mathbb{E}_t\hat{q}_{t+1} - \mathbb{E}_t\hat{r}_{t+1} \tag{18}$$

$$\hat{\pi}_t = \kappa\hat{y}_t + \beta\mathbb{E}_t\hat{\pi}_{t+1} \tag{19}$$

with

$$\mathbb{E}_t\hat{r}_{t+1} = \hat{i}_t - \mathbb{E}_t\hat{\pi}_{t+1} - \varrho$$
$$\hat{i}_t = \varrho + (1+\phi)\,\mathbb{E}_t\hat{\pi}_{t+1} + \varepsilon^i_t$$

where $\varrho \equiv \log r^*$, $\kappa \equiv \lambda(1+\varphi)$, and $\gamma \equiv \delta_1/[1+\delta_1-(1-\delta_2)^{\frac{1+\sigma}{\sigma}}]$ is the steady-state consumption share of retirees. Hats denote deviations from steady state (except for $\hat{i}_t$, which denotes the log of i_t).

From (16) one can see how the workers' Euler equation incorporates both the standard force of intertemporal substitution, as captured by the first RHS term, and a second term which captures wealth-related factors associated with retirement. As the probability of retiring (δ_1) goes up, the weight on wealth-related factors increases relative to intertemporal substitution. Greater retirement preoccupations thus imply that wealth-related factors will be more central to consumption decisions and the monetary transmission mechanism.

Note from (15) and (16) that wealth-related factors consist of two parts: an effect via the asset price, $\hat{q}_t$, and an effect stemming from the impact on retirees' MPC out of wealth, $\hat{\Gamma}_t$. Regarding the former, (18) shows that a higher expected rate path depresses the price q of the long-term bond. Via equations (15) and (16) this lowers consumption demand. We call this the "asset valuation channel". It works as a pure financial wealth effect and in the conventional direction, with rate hikes weighing on activity (see Caramp

to increase premiums during the zero-interest rate era, thus asking for greater saving efforts from their members. See, e.g., https://macrosynergy.com/research/low-for-long-rates-pressure-on-pensions-and-insurances/. As a concrete example, ABP (the largest Dutch pension fund) issued a statement back in 2019 (www.abp.nl/content/dam/abp/nl/documents/persbericht%20premie-indexatie%202020.pdf) saying "Pensions are becoming increasingly expensive [...] With the current pension ambition and the expectation that interest rates will remain low for a long time, higher premiums will be needed."

and Silva (2023) for a RANK model with this effect at play, also via the presence of long-term bonds).

When $\sigma > 1$, retirees' MPC out of wealth is positively related to the expected rate path, bringing a countervailing force. The reason is that, for a given value of assets, a higher rate path implies that these assets will come with a superior income stream. This greater income reduces the need to hold as many assets for retirement, thus lowering asset demand, stimulating goods demand. We call this the "asset demand channel". Since working households care about the retirement state when $\delta_1 > 0$, $\hat{\Gamma}_t$ shows up in (16) too (just weighted by the retirement probability δ_1). This channel works in the unconventional direction when $\sigma > 1$, with higher rates *boosting* activity.

4.2 How the effect of interest rates on activity varies along the yield curve

To see these effects differently, note that both q_t and Γ_t can be expressed as function of current and future interest rates – leading to a term structure representation for the Euler equation. Disregarding ε_t^{β} for a moment, the workers' Euler equation can be written as:

$$\hat{c}_t^w = (1 - \delta_1)\,\mathbb{E}_t \hat{c}_{t+1}^w - \frac{1}{\sigma}\mathbb{E}_t r_{t+1} + \delta_1 \sum_{j=1}^{\infty} \beta^j \left[\frac{\sigma - 1}{\sigma} (1 - \delta_2)^{\frac{j}{\sigma}} - (1 - \mu)^j \right] \mathbb{E}_t r_{t+1+j} \quad (20)$$

This formulation can be seen as incorporating several special cases present in the literature. For $\delta_1 = 0$, we obtain the standard RANK Euler equation. If $\sigma = 1$ and $\delta_1 > 0$, we have a formulation that is equivalent to putting assets directly into the utility function. Finally, with $\sigma = 1$, $\delta_1 > 0$, and $\mu = 1$, we have a discounted Euler equation. Note that if $\sigma \leq 1$ ($EIS \geq 1$), then interest rates *at all future horizons* enter this Euler equation with a negative sign. Interest rate policy then always works in the conventional way. Moreover, the more a rate decrease (increase) is viewed as being persistent, the more it will stimulate (contract) demand.

In contrast, when $\sigma > 1$ ($EIS < 1$), monetary policy can affect the economy very differently depending on whether it only affects short-term rates, or if interest rates further out in the term structure are affected. In the remainder of this paper, we will focus our discussion on the case where $EIS < 1$ (which, according to studies like Yogo (2005), Best et al. (2020), Ring (2024), and Crawley (2025), is the most empirically plausible case).

The first aspect to note from (20) is that a hike in the short-term rate r_{t+1} will always lower consumption (and vice versa for a cut). However, the effects of future rates on y_t will depend on the sign of $\left[\frac{\sigma-1}{\sigma}(1-\delta_2)^{\frac{j}{\sigma}}-(1-\mu)^j\right]$. This term captures the competition between valuation effects resulting from interest rate changes, versus the induced effects on asset demand (i.e., the desire to save for retirement).[17] Holding $\mathbb{E}_t\hat{c}^w_{t+1}$ constant, (20) shows that when σ is sufficiently high and/or the interest rate considered is sufficiently far into the future, a higher rate favors *more* consumption in the present. In other words, equation (20) shows that the partial effect of increasing interest rates on current consumption (holding $\mathbb{E}_t\hat{c}^w_{t+1}$ constant) will tend to change sign, from negative to positive, as one looks further in the future.[18] This arises as valuation effects only affect long-term assets, and these diminish further out in the future when $\mu > 0$ (which implies that the duration in assets is finite). Importantly, such sign-switching cannot arise under a discounted Euler equation formulation (more on this around Proposition 2 below).

However, (20) only provides a partial analysis since it is holding $\mathbb{E}_t\hat{c}^w_{t+1}$ constant and ignores retirees' consumption. Before deriving explicit expressions for the impact of future rates on current activity, we need to ensure that the equilibrium of the system (14)-(18) is well defined, i.e. stable and unique. Recall that monetary policy is governed by $(1+\phi)$, which expresses the degree to which expected interest rates are increased in response to expected inflation. The conventional Taylor principle suggests that we need $\phi > 0$. However, in our setup, the model maintains determinacy even if $\phi = 0$:

Proposition 1. *With $\theta > 0$ (sticky prices), a constant real rate policy ($\phi = 0$) is sufficient to deliver determinacy.*

Proofs of all propositions are in Appendix B. In light of Proposition 1, the rest of the paper will set $\phi = 0$ to ensure determinacy while simultaneously allowing us to discuss the effects of different real rate paths on activity (and see Appendix C for a visual representation of the model's determinacy region). Once we solve (15) and (16) forward, the impact of future rates on current activity and inflation can be expressed as:

$$\hat{y}_t = \sum_{j=0}^{\infty} \psi^y_j \mathbb{E}_t \hat{r}_{t+1+j} \tag{21}$$

$$\hat{\pi}_t = \sum_{j=0}^{\infty} \psi^\pi_j \mathbb{E}_t \hat{r}_{t+1+j} \tag{22}$$

[17] Note that asset duration is governed by $(1-\mu)$. The duration of pension-related liabilities is increasing in $(1-\delta_2)$, as the expected duration of the retirement state is decreasing in the death probability δ_2.

[18] This can be seen from the fact that $\beta^j[\frac{\sigma-1}{\sigma}(1-\delta_2)^{\frac{j}{\sigma}}-(1-\mu)^j]$ will be positive for high enough j as long as $\sigma > 1$ and $(1-\mu) < 1$, that is, under the condition that not all bonds are consols.

with $\psi_0^y = -\frac{1}{\sigma}$, $\psi_0^\pi = -\frac{\kappa}{\sigma}$,

$$\begin{aligned}\psi_j^y &= (1-\delta_1)\psi_{j-1}^y + \xi_j^\psi, \\ \psi_j^\pi &= \beta\psi_{j-1}^\pi + \kappa\psi_j^y,\end{aligned}$$

and

$$\xi_j^\psi \equiv \frac{\sigma-1}{\sigma}\left[\delta_1 - \gamma(1-\delta_1)\frac{1-\beta(1-\delta_2)^{\frac{1}{\sigma}}}{\beta(1-\delta_2)^{\frac{1}{\sigma}}}\right]\beta^j(1-\delta_2)^{\frac{j}{\sigma}} - \left[\delta_1 - \gamma(1-\delta_1)\frac{1-\beta(1-\mu)}{\beta(1-\mu)}\right]\beta^j(1-\mu)^j.$$

Here, each coefficient ψ_j^y represents the isolated impact that the real rate at horizon j has on output in the present (with ψ_j^π representing the equivalent concept for inflation). Note that it is always the case that an increase in the near-term rate $\mathbb{E}_t\hat{r}_{t+1}$ depresses activity, as this effect is driven solely by intertemporal substitution ($\psi_0^y = -\frac{1}{\sigma} < 0$). However, the effect of future interest rates becomes ambiguous as three forces are at play: intertemporal substitution, valuation effects, and effects on asset demand. Before deriving some of the properties of the ψ_j's when $\delta_1 > 0$ (i.e., when life-cycle forces are present), it is worth recalling that our model collapses to RANK for $\delta_1 = 0$. In that case, $\psi_j^y = -\frac{1}{\sigma}$ and $\psi_j^\pi = \kappa\frac{1-\beta^{j+1}}{1-\beta}\psi_j^y$ for all $j \geq 0$. This implies that near-term interest rates always have the exact same effect on output as rates further out into the term structure (with this effect always equal to $-\frac{1}{\sigma}$).

In contrast, as noted in Proposition 2, when $\delta_1 > 0$, the sign of ψ_j^y becomes dependent on the $EIS = 1/\sigma$. If the EIS is sufficiently large, rates at all horizons will have conventionally-signed effects as intertemporal substitution remains dominant. But when the EIS is sufficiently small, interest rates further out into the future will obtain the *unconventional* sign, since asset demand effects (driven by the interest income effect) will dominate.

Proposition 2. *For $\delta_1 > 0$ (i.e., when introducing retirement risk, giving rise to our "FLANK" model), we have that:*

(a) The ability of interest rates to affect activity and inflation in the conventional direction (i.e., with contractionary shocks lowering activity and inflation, and vice versa) is weakened relative to RANK: $\psi_j^y > -\frac{1}{\sigma}$ and $\psi_j^\pi > -\frac{\kappa}{\sigma}\frac{1-\beta^{j+1}}{1-\beta}$, for all $j \geq 1$;

(b) In the limit, taking the horizon j to infinity, $\mathbb{E}_t\hat{r}_{t+1+j}$ ceases to affect activity and inflation in the present: $\lim_{j\to\infty}\psi_j^y = 0$ and $\lim_{j\to\infty}\psi_j^\pi = 0$;

(c) At every horizon $j \geq 1$, ψ_j^y and ψ_j^π are increasing in σ; they eventually become positive as σ is increased;

(d) The ability of interest rate policy to affect activity and inflation in the conventional direction is increasing in retirees' death probability (δ_2) and increasing in the duration of available assets (i.e., decreasing in μ) for all $j \geq 1$.

The key takeaway from Proposition 2 is that, with life-cycle forces, the effect that interest rates have on activity can vary along the yield curve – both quantitatively and qualitatively. In our FLANK setup, higher near-term rates can be contractionary ($\psi_j^y < 0$ for $j < \widetilde{j}$), whereas simultaneously higher rates further out into the term structure can be expansionary ($\psi_j^y > 0$ for $j > \widetilde{j}$). Figure 2 illustrates this by plotting how ψ_j^y evolves as the horizon j lengthens. As the dashed lines show, $\psi_j^y = -1/\sigma \ \forall j$ in RANK, whereas the solid lines convey the more involved forces present in FLANK.

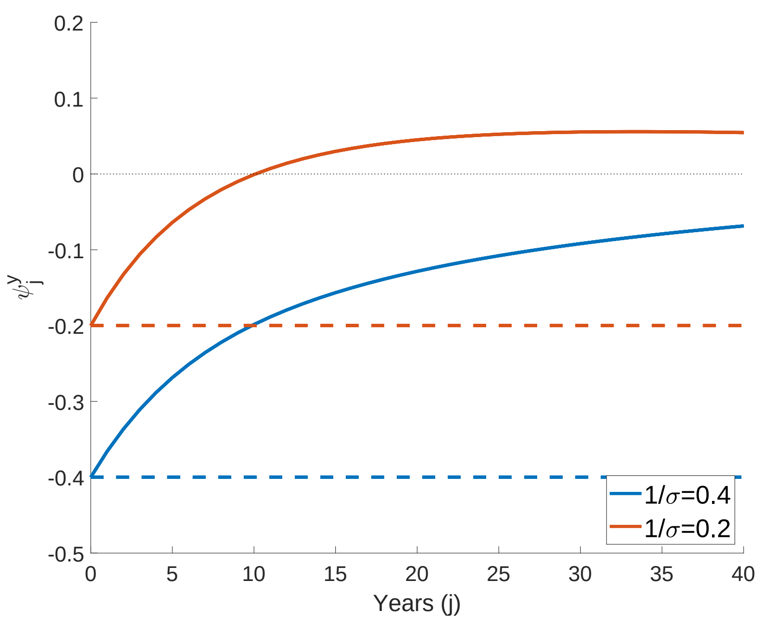

Figure 2: Evolution of ψ_j^y-coefficients along the yield curve in FLANK (solid) and RANK (dashed).

Parts (a) and (b) of Proposition 2 are shared by models with a discounted Euler equation (McKay et al., 2017). Parts (c) and (d) are specific to FLANK. Part (c) implies that, in FLANK, interest rates further out in the yield curve may have *opposite* effects to that of near-term rates, with higher long-term rates *boosting* activity. This is something that can neither arise in RANK, nor under a discounted Euler equation. As discussed in Appendix D, this prediction of FLANK is consistent with the empirical observation that an inverted yield curve is often followed by an economic slowdown – with our model suggesting a causal link.

Part (d) of Proposition 2 provides additional insight on the ψ_j coefficients. It shows that interest rate policy loses potency in the conventional direction as households' expected time spent in retirement increases (lower δ_2). This increases the duration of household liabilities – with them having to finance a longer consumption stream in retirement, where households rely on asset income – meaning that low future rates (which are normally expansionary) incite more savings by working households and slower asset depletion by retirees.

Part (d) also implies that interest rate policy loses potency in the conventional direction when asset duration decreases (higher μ). The reason is that this weakens the asset valuation effect, which works in the usual direction (with lower rates being expansionary). This is relevant in considering how QE might affect the monetary transmission mechanism. Since it acts like an asset swap, with the central bank replacing high-duration assets (long-term bonds) with overnight central bank reserves of zero duration, QE can be seen as the central bank pushing up μ (lowering the share of long-term bonds held by the public[19]). This makes the economy less interest rate sensitive – rendering conventional monetary policy (conducted via the interest rate) less potent.[20]

It important to emphasize that Part (c) of Proposition 2 is central to our key results which are to follow, as it implies that persistent rate changes may have *qualitatively different* effects compared to more temporary ones.

4.3 Effect of interest rate persistence on potency and direction

We are now able to discuss how the potency of monetary policy shocks can change with their persistence. Consider a shock ε_t^i to the interest rate rule that follows an AR(1) process with autocorrelation parameter ρ_i (as specified in (12)). This implies that the policy shock induces a time path for the real rate given by $\mathbb{E}_t\hat{r}_{t+1+j} = \mathbb{E}_t\varepsilon^i_{t+j} = (\rho_i)^j\epsilon^i_t$. The impact responses to such monetary policy shock are:

[19] At this stage it is important to note that our Blanchard-Yaari-Gertler setup implies that Barro's (1974) Ricardian Equivalence does not hold; because of this breakdown, the maturity structure of assets held by the public starts to matter. For $\delta_1 = 0$, Ricardian Equivalence holds and μ no longer matters for (21) and (22).

[20] Concerns related to this aspect of our model have recently come to the fore. As noted in Bloomberg (2023): "UK households are on aggregate about £10 billion ($12.7 billion) a year better off as a result of a jump in interest rates [...] At current rates, savers collectively are earning £24 billion more a year than in November 2021 [...] Respondents to GfK's June consumer confidence barometer said their personal finance situation had improved sharply last month, despite the surge in mortgage rates [...] The data suggests interest rates may not be as effective a monetary policy tool as they were in 2008".

$$\hat{y}_t = \Psi^y(\rho_i)\epsilon_t^i, \tag{23}$$
$$\hat{\pi}_t = \frac{\kappa}{1-\rho_i\beta}\Psi^y(\rho_i)\epsilon_t^i, \tag{24}$$

where

$$\Psi^y(\rho_i) \equiv \sum_{j=0}^{\infty}\psi_j^y\rho_i^j$$
$$= -\frac{1}{\sigma}\frac{(1-\gamma)(1-\delta_1)}{1-\rho_i(1-\delta_1)} + \left[\gamma + \frac{\delta_1(1-\gamma)}{1-\rho_i(1-\delta_1)}\right]\left[\frac{\frac{\sigma-1}{\sigma}}{1-\rho_i\beta(1-\delta_2)^{\frac{1}{\sigma}}} - \frac{1}{1-\rho_i\beta(1-\mu)}\right] \tag{25}$$

captures the effect of a monetary policy shock ϵ_t^i with persistence ρ_i on current output. Since our model features no state variables, we have that $\hat{y}_t = \rho_i^t\hat{y}_0$ and $\hat{\pi}_t = \rho_i^t\hat{\pi}_0$ – implying that results continue to apply at all horizons $t \geq 0$.

Equations (23) and (24) have several interesting implications for how a shock's persistence affects potency. If either $\delta_1 = 0$ (no retirement preoccupations) or $\sigma \leq 1$, then more persistent monetary policy shocks always have greater potency than temporary ones. In particular, when persistence ρ_i goes to 1, the potency of monetary shocks becomes very large, and goes to infinity if $\delta_1 = 0$ (i.e., in RANK). It is because of this potency that it is generally thought that monetary policy cannot keep real rates away from their flexible-price counterpart r^* for long periods without having major effects on inflation. However, in the presence of a retirement savings motive ($\delta_1 > 0$) and if $\sigma > 1$, the link between the persistence of monetary shocks and their effect on the economy becomes more involved.

While (23) and (24) show that the link between the persistence of monetary shocks and their effects on the economy depends on many parameters, Proposition 4 emphasizes the role played by the EIS $(1/\sigma)$. In particular, it emphasizes the existence of two threshold levels for σ for which the relationship between monetary shock persistence and their effect on the economy changes qualitatively.

Proposition 3. *For $\delta_1 = 0$, $\Psi^y(\rho_i) < 0$ for all $\rho_i \in [0,1]$, $\partial\Psi^y(\rho_i)/\partial\rho_i < 0$, and $\lim_{\rho_i\to 1}\Psi^y(\rho_i) = -\infty$.*

Proposition 4. *If $\delta_1 > 0$, then $\lim_{\rho_i\to 1}\Psi^y(\rho_i)$ is finite and $\exists\sigma^*, \sigma^{**}$ with $\sigma^{**} > \sigma^*$, such that for very persistent monetary policy shocks (ρ_i close to 1):*

(a) If $\sigma < \sigma^*$, then $\Psi^y(\rho_i) < 0$ and $\partial\Psi^y(\rho_i)/\partial\rho_i < 0$, meaning that more persistent

shocks have a stronger effect on activity in the conventional direction (i.e., with rate-increasing shocks lowering activity and vice versa);

(b) If $\sigma > \sigma^*$, then $\partial \Psi^y(\rho_i)/\partial \rho_i > 0$, meaning that increases in shock persistence DECREASE *the shock's effect on activity in the conventional direction;*

(c) If $\sigma > \sigma^{**}$, then $\Psi^y(\rho_i) > 0$, meaning that sufficiently persistent monetary policy shocks affect activity in the unconventional direction.

The main aspect to note from Proposition 4(b) is that, when σ is high enough in FLANK, a more persistent monetary shock will be *less* potent than a more temporary one – giving a stark contrast with RANK (covered by Proposition 3).[21] This "persistence-potency trade-off" arises because the effects of monetary shocks on consumption are not just driven by intertemporal substitution in FLANK. Instead, they are also shaped by how the rate change affects the desire to accumulate, and hold on to, assets (to ensure consumption in retirement). The latter depends on whether the lower (higher) rates are incentivizing households to hold more (less) wealth and whether valuation effects are sufficiently large to offset any changes in their desire to save. Proposition 4 shows that as σ increases, intertemporal substitution becomes less relevant and the impact on asset demand will eventually dominate the valuation effect. This then causes more persistent shocks to have less of an effect on activity than more temporary changes.[22] Interestingly, such a pattern has been observed by various empirical studies, including Uribe (2022), McKay and Wolf (2023, their Appendix C.2), Miescu (2023), Swanson (2024), and Braun et al. (2025); our Appendix D provides further evidence.

In fact, the operation of monetary policy can even flip sign, as implied by part (c) of the proposition. To visualize this, Figure 3 plots $\Psi^y(\rho_i)$ as ρ_i varies between 0.5 and 1.[23] The figure illustrates that, for rather transitory shocks, life-cycle forces do not affect the monetary transmission mechanism much (i.e., FLANK behaves much like RANK for low ρ_i). But as ρ_i increases, the two models diverge: whereas RANK implies that very persistent shocks are incredibly potent in the conventional direction (with this potency

[21] It is also possible to show that $\Psi^y(\rho_i)$ is decreasing in δ_2 and increasing in μ, which would be another way to state the message conveyed by part (d) of Proposition 2.

[22] This result is reminiscent of Lucas and Rapping (1969), who show that the response of labor supply may vary with the persistence of the wage impulse. When the latter is rather transitory, the substitution effect is likely dominant – making labor supply increase with the wage rate. But when the wage changes in a rather persistent manner, the income effect gains importance – potentially causing labor supply to fall with wages.

[23] This figure was generated using the following calibration at the annual frequency: $\sigma = 4$, $\beta = 0.96$, $\delta_1 = 1/45$ (an expected working life of 45 years), $\delta_2 = 1/20$ (an expected retired life of 20 years), and $\mu = 0.15$ (average bond maturity of 6.7 years).

going to infinity in the limit), FLANK suggests the opposite may arise – with $\Psi^y(1)$ close to zero (or positive) being a plausible outcome.

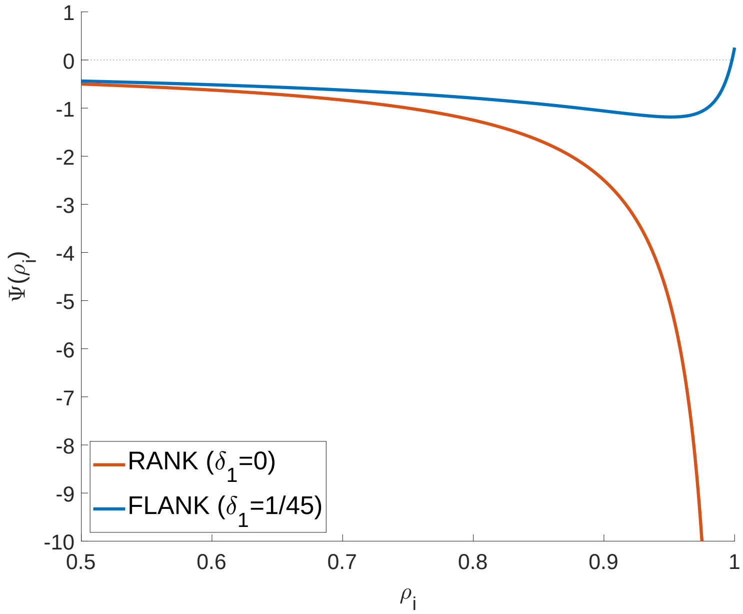

Figure 3: $\Psi^y(\rho_i)$ in RANK and FLANK. Other parameters calibrated as in footnote 23.

4.4 The effect of (near-)permanent monetary policy shocks

At this stage, one might wonder: what does FLANK imply for a central bank's ability to keep its policy rate away from the natural rate r^* on a lasting basis (thereby affecting longer-term rates via the expectations theory of the term structure)? This is like asking whether the central bank can conduct monetary policy via an interest rate rule with an intercept (the long-run average rate the central bank is aiming for) different to the true r^*, without creating massive inflation or deflation.

The consequences of keeping r permanently away from r^* can be understood by considering the effects of a permanent monetary policy shock.[24] While our log-linearized model is not formally equipped to analyze truly permanent ($\rho_i = 1$) shocks, it is insightful to consider the analytical expression for Ψ^y that results in the limit where $\rho_i \to 1$ in equation (25). One then obtains:

[24] Agents in our model only care about the interest rate path, not about the decomposition between systematic monetary policy (including the intercept) and shocks to that rule (McKay and Wolf, 2023).

$$\Psi^y(1) = \sum_{j=0}^{\infty} \psi_j^y = \underbrace{-\frac{1}{\sigma}\frac{(1-\gamma)(1-\delta_1)}{\delta_1}}_{intertemporal\ substitution} + \underbrace{\frac{\sigma-1}{\sigma}\frac{1}{1-\beta(1-\delta_2)^{\frac{1}{\sigma}}}}_{asset\ demand} - \underbrace{\frac{1}{1-\beta(1-\mu)}}_{asset\ valuation}. \tag{26}$$

Written this way, the decomposition central to our paper becomes clear. The first term captures intertemporal substitution, as in the standard Euler equation. It is always negative and goes to zero as $\frac{1}{\sigma} \to 0$. The second term captures the asset demand effect. This is primarily driven by the duration of household liabilities, as governed by the death probability δ_2, which determines the expected length of retirement (where the household still wants to consume, but only enjoys interest income).[25] When $\sigma > 1$ this term is positive, being a force working in the unconventional direction. The third term captures the asset valuation effect, driven by μ (asset duration). Whenever the sum of the last two terms is positive, meaning that the duration in the household's asset portfolio is not enough to compensate for its negative duration gap stemming from the need to finance consumption in retirement, the total effect $\Psi^y(1)$ could be close to zero (as the first term in (26) is negative). In the special case where $\frac{1}{\sigma} \to 0$ and $\mu \to 0$, we have $\Psi^y(1)$ *exactly* equal to 0. This arises because consumption then equals the flow value of wealth, $(r-1)a$, while the value of wealth itself is proportional to $\frac{1}{r-1}$.

Equation (26) also allows for an insightful reinterpretation, distinguishing between just two forces. Combining the effects relating to intertemporal substitution and asset demand, one obtains a term that captures the effect of a permanent rate increase on the economy's average MPC out of financial wealth ($MPCoW$).[26] $\Psi^y(1)$ then reads:

$$\Psi^y(1) = \underbrace{-\frac{1}{\sigma}\frac{(1-\gamma)(1-\delta_1)}{\delta_1} + \frac{\sigma-1}{\sigma}\frac{1}{1-\beta(1-\delta_2)^{\frac{1}{\sigma}}}}_{average\ MPC\ out\ of\ financial\ wealth\ (MPCoW)} - \underbrace{\frac{1}{1-\beta(1-\mu)}}_{asset\ valuation}. \tag{27}$$

Since the asset valuation effect always works in the conventional direction (higher rates depress demand), equation (27) implies that $\Psi^y(1) \approx 0$ is possible if the $MPCoW$ rises sufficiently in response to a permanent rate increase. In FLANK, this can easily happen

[25]The asset demand effect is maximized for $1/\sigma \to 0$. In that limit, the household is infinitely risk averse, meaning that it only consumes its interest rate income – never daring to touch the principal (including any capital gains) for fear of outliving assets. This actually seems a reasonable approximation to the observed behavior of retirees, who do not dissave much in retirement; recall footnote 4.

[26]Averaging is over workers and retirees. The effect on the $MPCoW$ for retirees is given by $\frac{\sigma-1}{\sigma}\frac{1}{1-\beta(1-\delta_2)^{\frac{1}{\sigma}}}$, while that on working households equals $\frac{\sigma-1}{\sigma}\frac{1}{1-\beta(1-\delta_2)^{\frac{1}{\sigma}}} - \frac{1}{\sigma}\frac{1-\delta_1}{\delta_1}$. Since $\frac{1}{\sigma}\frac{1-\delta_1}{\delta_1} > 0$, the effect of r on the $MPCoW$ of working households is less positive (or more negative) than that on retirees – the reason being that interest income is less important to the former group.

because agents have less need to hold assets, to maintain a given retirement consumption stream, when assets generate a higher return.[27,28]

Ultimately, the above discussion concerns a quantitative question. For the calibration used in Figure 3, $\Psi^y(1)$ is indeed close to zero. But since there is uncertainty regarding the appropriate calibration, Figure 4 goes further and presents a heatmap for $\Psi^y(1)$.[29] It conveys the different values taken on by $\Psi(1)$ for different values of the $EIS(=\frac{1}{\sigma})$ and bond duration, as governed by μ. For the other parameters, of which there are only three, we fix $\delta_2 = 1/45$ (an expected working life of 45 years), $\delta_2 = 1/20$ (an expected retirement span of 20 years), and set $\beta = 0.96$.

Our biggest challenge relates to the plausible range for the EIS. To this end, we draw from Best et al. (2020) which uses a frontier empirical strategy. Their preferred EIS estimate lies close to zero (at around 0.1). At the other end, they report values up to 0.3 (see their Table 3B, pooled estimate), so we go up to 0.35 to be inclusive of higher values (which are also consistent with Havránek's (2015) meta-analysis, which reports estimates centered around 0.3-0.4). With respect to average bond maturities, we consider values above 5 years (i.e., $\mu \leq 0.2$), which is aimed at capturing a set of interpretations for assets held. Lower durations (higher μ) are appropriate when only thinking of government bonds (which, in the U.S., have an average duration of just over 5 years); higher durations (lower μ) are reasonable when thinking of a combination of bonds, equity, and real estate.[30] We aim to be quite inclusive in the range of parameters explored, to give a sense of the possible outcomes in FLANK.

In Figure 4, blue areas represent negative values (this is the "conventional" region, as we are considering a permanent rate hike); red areas represent *positive* values for $\Psi^y(1)$.

[27] $\partial MPCoW/\partial r > 0$ also aligns with the findings in Lettau and Ludvigson (2001), who present evidence from asset prices suggesting that household consumption increases with (expected) returns.

[28] It may be helpful to consider the steady-state consumption equation that lies behind $\Psi(1)$: $C = \beta^{\frac{-1}{\sigma}} \left[r^{\frac{\sigma-1}{\sigma}} - [(1-\delta_2)\beta]^{\frac{1}{\sigma}} \right] \left[\chi(1-\beta(1-\delta_1)r)^{\frac{1}{\sigma}} \delta_1^{\frac{-1}{\sigma}} + (1-\chi)(1-\delta_1)^{\frac{1}{\sigma}} \right] a$, where χ is the steady-state share of assets held by workers. As the EIS $(1/\sigma)$ goes to zero, this expression simplifies to $C = (r-1)a$, so that consumption equals the flow value of financial assets. This may appear similar to the permanent income hypothesis, but is quite different as in our setup $C = (r-1)a$ is a general equilibrium relationship when activity is demand determined. In addition, "a" represents only financial assets and does not include the value of human capital (which is endogenous in FLANK). Recall from Figure 1 that we found consumption to be highly correlated with the flow value of wealth, $(r-1)a$, but not with wealth itself. Seen through our model, this suggests a very low EIS.

[29] Since our model is log-linearized, it is not formally equipped to handle fully permanent shocks, which is why Figure 4 is generated with $\rho_i = 0.99$. The same applies to Figure 5.

[30] Weber (2018) puts the duration of the S&P 500 at around 20 years. The duration of housing is estimated to be around 8 years (Burgert et al., 2024). In the Fed's FRB/US model, a highly persistent unit monetary policy shock changes household aggregate wealth holdings by approximately 10%, which corresponds to $\mu = 0.1$ in our FLANK model.

White areas indicate that $\Psi^y(1)$ is close to zero, with black lines marking iso-$\Psi^y(1)$ curves. An iso-$\Psi^y(1)$ curve of $\pm 1\%$ implies that a permanent real rate increase by 1 percentage point relative to r^*, would cause an output gap of 1%. With a standard Euler equation (including the discounted variant), the whole area would be dark blue. In particular, under RANK the entire surface would be valued at $-\infty$. In contrast, Figure 4 shows that positive values for $\Psi^y(1)$ are about as plausible as negative ones. FLANK thus gives little reason to believe that persistently low (high) interest rates are more likely to stimulate (depress) the economy, than to depress (stimulate) it. This, by itself, is an important take-away.[31]

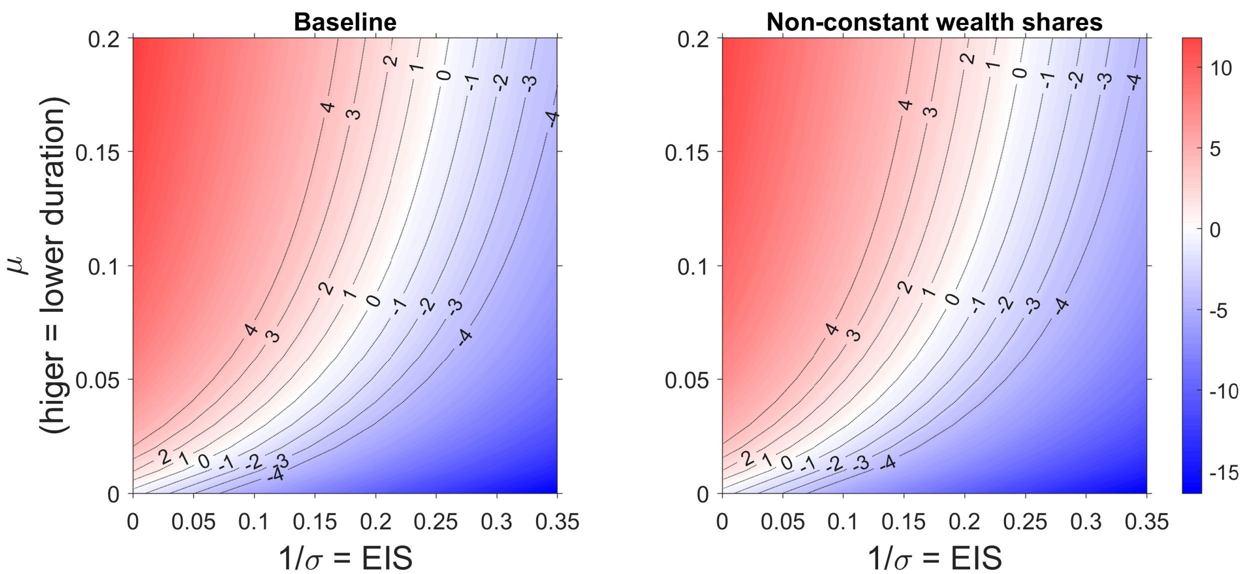

Figure 4: $\Psi^y(1)$ as a function of σ and μ in FLANK. The left panel shows this for the baseline model (featuring a transfer scheme to keep the wealth shares constant at their steady state), while the right panel does not impose this simplifying assumption. Other parameters are calibrated as in footnote 23.

The area where $\Psi^y(1)$ is *exactly* equal to zero, is of measure zero – making it not very relevant. Nonetheless, the figure shows that there is a considerable area where $\Psi^y(1)$ may be considered quite small. Recall that over the period from 1990 to 2019, the U.S. output gap varied by several percentage points without inflation moving much. This suggests that, when inflation expectations are well anchored, an output gap of a few percentage points might not affect inflation by a lot. Accordingly, Figure 4 hosts a considerable region where a permanent departure of real rates from r^* could be consistent

[31] While much of our discussion focuses on the idea that $\Psi^y(1)$ may be close to zero, it is worth noting that the possibility of $\Psi^y(1)$ being positive (instead of negative) suggests that low-for-long policies may have contributed to depressing the economy instead of stimulating it.

with inflation remaining close to target.

Figure 4 thus illustrates that the effect of real rates permanently deviating from r^* is both qualitatively and quantitatively quite different in FLANK, relative to a more standard New Keynesian model. In particular, in FLANK the effect can be positive, negative or close to zero – as opposed to always negative. While we think that the most intriguing take-away from this figure is that $\Psi^y(1)$ could be close to zero, one may wonder how robust this finding is with respect to modifications of our model. Section 6 discusses this in some detail, but one may already recall that the above results were derived under the assumption that the relative wealth share of retired versus working households was held constant via a tax-transfer scheme. Does this affect the properties of $\Psi^y(1)$? Figure 4's right panel, which shows the same heatmap *without* the simplifying assumption (in which case we can still solve the model numerically), points to robustness: both the qualitative and quantitative features of $\Psi^y(1)$ are virtually unchanged.

4.5 Monetary versus demand shocks in FLANK

Another interesting feature of FLANK is that it breaks the equivalence (for example present in RANK) between monetary shocks and other types of demand shocks. Here, we illustrate this point by considering shocks to the discount rate (recall equation (13)), but the point is more general. To see this, observe that there exists an equivalent representation to equations (21)-(22), which were derived under $\varepsilon_t^\beta = 0$, when allowing for discount rate shocks. In particular, Appendix B shows that the effects of discount rate shocks "ε_t^β" on output are given by:

$$\hat{y}_t = \sum_{j=0}^{\infty} \omega_j^y \mathbb{E}_t \varepsilon_{t+j}^\beta \tag{28}$$

$$\hat{\pi} = \sum_{j=0}^{\infty} \omega_j^\pi \mathbb{E}_t \varepsilon_{t+j}^\beta \tag{29}$$

with $\omega_0^y = -\frac{1}{\sigma}$, $\omega_0^\pi = -\frac{\kappa}{\sigma}$,

$$\omega_j^y = (1-\delta_1)\omega_{j-1}^y + \xi_j^\omega,$$

$$\omega_j^\pi = \beta\omega_{j-1}^\pi + \kappa\omega_j^y, \text{ and}$$

$$\xi_j^\omega \equiv -\frac{1}{\sigma}\left[\delta_1 - \gamma(1-\delta_1)\frac{1-\beta(1-\delta_2)^{\frac{1}{\sigma}}}{\beta(1-\delta_2)^{\frac{1}{\sigma}}}\right]\beta^j(1-\delta_2)^{\frac{j}{\sigma}}.$$

Crucially, with $\delta_1 > 0$, the coefficients on the discount rate shock at each horizon $j > 0$ are *not* proportional to those for the monetary policy shock. For RANK ($\delta_1 = 0$) the coefficients *are* proportional. In that case, a monetary policy shock induces the exact same dynamics as a discount rate shock – meaning that the former is extremely well-suited to offset the latter. However, in FLANK that is no longer true: while discount rate shocks continue to operate via intertemporal substitution, policy-induced shocks to the interest rate are "special" as they come with an offsetting effect (changes in interest income affecting asset demand) that give rise to a persistence-potency trade-off. Note that the time-t impact of a persistent AR(1) discount rate shock is given by:

$$\Omega^y(\rho_\beta) \equiv \sum_{j=0}^{\infty} \omega_j^y \rho_\beta^j$$
$$= -\frac{1}{\sigma}\frac{(1-\gamma)(1-\delta_1)}{1-\rho_\beta(1-\delta_1)} - \frac{1}{\sigma}\frac{\gamma + \frac{\delta_1(1-\gamma)}{1-\rho_\beta(1-\delta_1)}}{1-\rho_\beta\beta(1-\delta_2)^{\frac{1}{\sigma}}}$$

From this, it is easy to see that $\Omega^y(\rho_\beta) < 0$ for all $\rho_\beta \in [0,1]$, with $\partial\Omega^y(\rho_\beta)/\partial\rho_\beta < 0$ (meaning that more persistent shocks are more potent in the conventional direction).

These findings suggest that monetary policy is less well equipped to offset demand shocks in FLANK, especially when demand shocks are very persistent.

5 Reflections on r*

FLANK implies that the effects of interest rates on activity will vary along the yield curve, likely switching sign along the way. This section will show that this has important implications for the relevance of the natural rate (r^*) as a policy anchor, and for r^* estimation. In particular, our setup implies that precise knowledge of r^* may not be very important for inflation-targeting central banks – as the system may be very "forgiving" to the central bank working with a biased value for r^*. This indicates that central banks might still be able to fulfill their mandate in a satisfactory way, even if they are ill-informed about the true value of r^*. In addition, we will show that a common method used to infer r^* may be biased and essentially deliver the central bank's own prior beliefs.

5.1 The (ir)relevance of r*

Standard models suggest that the location of r^* is crucial for central banks to be aware of, since keeping rates away from that level for too long is bound to force inflation away from target.[32] In contrast, FLANK suggests that central banks may be much less constrained by r^*, potentially making r^* quasi-irrelevant and opening the door for monetary policy to influence longer-term real rates. To further clarify the extent to which monetary policy is constrained by r^*, consider the class of models where activity $\hat{y}_t$ can be related to the future path of interest rates via:

$$\hat{y}_t = \sum_{j=0}^{\infty} \psi_j^y \mathbb{E}_t(r_{t+1+j} - r^*).$$

As previously noted, this formulation (also shown in equation (21)) hosts the standard RANK model as well as our FLANK setup – with the models differing only with respect to implied coefficients for ψ_j^y.

Now consider a central bank which misperceives r^*, where we denote the central bank's perception of r^* by r^L (which can be seen as the central bank's long-run target for r). Would this misperception be problematic? The determination of output is now given by:

$$\hat{y}_t = \sum_{j=0}^{\infty} \psi_j^y \mathbb{E}_t(r_{t+1+j} - r^L) + \Psi^y(1)(r^L - r^*),$$

where $\Psi^y(1) \equiv \sum_{j=0}^{\infty} \psi_j^y$. This shows that the relevance of r^* for $\hat{y}_t$ depends crucially on the value of $\Psi^y(1)$. When activity is determined by a standard representative agent Euler equation, $\Psi^y(1) = -\infty$. In this case, making sure that r^L equals r^* is *absolutely crucial* for monetary authorities as deviations of r^L from r^* would have huge implications for activity and consequently inflation.[33]

However, in FLANK, $\Psi^y(1)$ may actually be close to *zero*. Deviations of r^L from r^* then do not affect activity much. And if the Phillips curve is not very steep, as for example argued by Hazell et al. (2022), an $(r^L - r^*)$-gap could have only a small effect on inflation. Therefore, when $\Psi^y(1)$ is small, a central bank could potentially adopt a policy rule where its long-term anchor for real rates r^L is substantially different from the true r^* without causing any major economic disruption.

In this sense, knowing r^* becomes quasi-irrelevant for the conduct of monetary policy, as the system is very forgiving to the central bank working with a biased r^*-belief. In the special case where $\Psi^y(1)$ is *exactly* zero, r^* becomes indeterminate and the central

[32]This notion also appears to be gaining popularity in practice, with the number of central bank speeches referring to the "natural/neutral interest rate" having risen sharply since 2015 (Borio, 2021).

[33]This logic captures why central banks are often thought to be heavily constrained by r^*, while it also explains why there is a Forward Guidance puzzle (Del Negro et al., 2013).

bank can set its long-term goal r^L freely, without any direct implications for output and inflation. Still, the choice for r^L will affect asset prices.

5.2 Estimation of r*

FLANK also has important implications for r^* estimation. To see this, note that a very typical formulation (sitting at the core of many popular DSGE models) for the consumption Euler equation reads:

$$\hat{c}_t = \alpha \mathbb{E}_t \hat{c}_{t+1} - \frac{1}{\sigma} \mathbb{E}_t (r_{t+1} - r^*_{t+1}) + v_t, \tag{30}$$

where the parameter $\alpha \leq 1$ reflects a generalization which allows the Euler equation to be discounted, and v_t represents a stationary demand shock. For illustration, we assume that v_t is an i.i.d. disturbance and that r^* follows a random walk: $r^*_{t+1} = r^*_t + w_t$, where w_t is again i.i.d.

If the data are thought to be driven by such an Euler equation, the work by Laubach and Williams (2003; "LW") offers a way to estimate r^*_t. In essence, it consists of creating an observation variable $z_t \equiv (\hat{c}_t - \alpha \mathbb{E}_t \hat{c}_{t+1} - \frac{1}{\sigma} r_t)\sigma$. Given this definition, which gives $z_t = r^*_{t+1} + \sigma v_t$, any long-run variation in z_t will be driven by r^*_{t+1} – implying that one can apply the Kalman filter to the z_t series and successfully recover an estimate of r^*_{t+1}.[34]

We now explore what this approach would uncover if the data were actually generated by the FLANK model, but it was misinterpreted as coming from a more standard Euler equation. In particular, we want to examine the case where one *thinks* the consumption data are generated by (30), but the actual data come from FLANK:

$$\hat{c}_t = \sum_{j=0}^{\infty} \psi^y_j \mathbb{E}_t r_{t+1+j} + \Psi^y(1) r^*_{t+1} + v_t, \tag{31}$$

with ψ^y_j as in (21). Combining (31) with an interest rate rule of the form:

$$i_t - \mathbb{E}_t \pi_{t+1} = \mathbb{E}^{CB}_t r^*_{t+1} + \varepsilon^i_t, \tag{32}$$

where $\mathbb{E}^{CB}_t r^*_{t+1}$ represents the central bank's perception of r^*_{t+1} (also following a random walk), we can again create $z_t = (\hat{c}_t - \alpha \mathbb{E}_t \hat{c}_{t+1} - \frac{1}{\sigma} r_{t+1})\sigma$ as suggested by LW. But in this case z_t will no longer be a noisy reflection of r^*_{t+1} only, as it is now given by:

[34] Throughout this section, we give the LW methodology its best chance by assuming that the central bank knows the private expectation $\mathbb{E}_t \hat{c}_{t+1}$. However, similar results arise if we assume that the central bank approximates this expectation with $\hat{c}_{t-1}$.

$$z_t = \sigma \left[\left(\frac{1}{\sigma} - \Psi^y(1)(1-\alpha) \right) \mathbb{E}_t^{CB} r_{t+1}^* + \Psi^y(1)(1-\alpha) r_{t+1}^* \right] + (\sigma - 1) v_t. \tag{33}$$

Equation (33) shows that z_t will only be a noisy reflection of r_{t+1}^*, uncontaminated by the central bank's own belief $\mathbb{E}_t^{CB} r_{t+1}^*$, when $\Psi^y(1) = \frac{-1}{(1-\alpha)\sigma}$. But $\Psi^y(1) = \frac{-1}{(1-\alpha)\sigma}$ only arises if the data are actually generated by an Euler equation of the form (30). Whenever $\Psi^y(1) \neq \frac{-1}{(1-\alpha)\sigma}$ (as is the case for FLANK; recall (26)), z_t will in part end up reflecting variations in the central bank's own perceptions $\mathbb{E}_t^{CB} r_{t+1}^*$. If $\Psi(1)$ is close to zero, then z_t will *mainly* reflect $\mathbb{E}_t^{CB} r_{t+1}^*$ instead of the true r_{t+1}^*.

Matters only get worse if one were to specify a more general interest rate rule. In particular, consider replacing (32) by:

$$i_t - \mathbb{E}_t \pi_{t+1} = \mathbb{E}_t^{CB} r_{t+1}^* + \phi_v v_t + \varepsilon_t^i,$$

where $\phi_v > 0$ allows the central bank to respond to demand shocks v_t. We then get:

$$z_t = \sigma \left[\left(\frac{1}{\sigma} - \Psi^y(1)(1-\alpha) \right) \mathbb{E}_t^{CB} r_{t+1}^* + \Psi^y(1)(1-\alpha) r_t^* \right] + \left[(\sigma - 1) + \phi_v \right] v_t.$$

Now, "$\phi_v v_t$" shows up in z_t, implying that the central bank's perception of r_t^* starts to co-move with its own short-term actions in response to demand shocks v_t. While standard logic suggests that any co-movement between a central bank's policy rate and r^*-estimates is due to the central bank successfully tracking the latter, our results imply that causality may run the other way: an initial negative, transitory demand shock ($v_t < 0$), which induces the central bank to cut its policy rate, might ignite a dynamic that leads the central bank to lower its estimate of r^* – which then has the unintended consequence of giving the initial rate cut more persistence through an unanticipated downward revision in the intercept of the policy rule (32). If $\Psi^y(1) \approx 0$, persistent rate changes don't affect activity and inflation much, meaning that there is no strong feedback from the system and hence no strong force pulling the central bank back towards the true r^* (recall Section 5.1).[35] $\mathbb{E}_t^{CB} r_{t+1}^*$ then obtains a self-fulfilling aspect – making it rational for markets to pay attention to the central bank's belief on r^*, even if markets do not think that the central bank has private information regarding r^*.

[35] John H. Williams (1931) famously argued that "The natural rate is an abstraction; like faith, it is seen by its works. One can only say that if the bank policy succeeds in stabilizing prices, the bank rate must have been brought in line with the natural rate, but if it does not, it must not have been." Our FLANK model suggests that these "works" might be rather weak, implying that there is not much to be learned from outcomes.

6 Discussion: assumptions and extensions

6.1 Assumptions

In this section we discuss various assumptions underlying our model. We will point out why our current assumptions could be easily relaxed and why they would not likely change our key insights. We also discuss how our results should be seen as "local", placing implicit bounds on how far interest rates could deviate from the true r^*.

Hand-to-mouth agents. Our model treats all households as intertemporal optimizers. This may seem inappropriate given the evidence supporting the presence of hand-to-mouth (HtM) consumers (Kaplan et al., 2014). Accordingly, the mechanisms in our model may appear relevant only for the financially well-off. We concur with this, but do not view it as a drawback for two reasons. First, the well-off account for much of total consumption demand – making them a natural focal point (in U.S. data, the wealthiest 20% account for nearly half of total consumption; Abbott and Brace, 2020). Second, one of the main insights from the HtM literature is that the dynamics of aggregate activity will primarily be driven by the behavior of optimizing households – even if the later are only a fraction of the total population (Werning, 2015). With HtM households, the decisions of optimizers are transmitted to wider economy via the non-optimizing households – potentially yielding amplification (Bilbiie, 2020; 2024), though recent micro-data from Norway fails to provide evidence for such amplification (Bilbiie et al., 2025). Either way, as long as the fraction of income going to HtM households is relatively stable, treating the economy as if solely driven by optimizers is a decent approximation. This is the interpretation we favor, recognizing that the modelled behavior may only reflect a subset of the population. While our model's structure is flexible enough to add HtM households, we choose not to – as this would complicate the setup without adding anything new.

Bequests. While our FLANK model does not include a bequest motive, we believe that its insights should carry through and may even be strengthened with such an extension. Bequest motives would likely accentuate the asset demand force present in FLANK. If parents not only care about the value of any assets they pass on, but also about the expected rate of return, a simple modelling approach is to think of bequests as consumption past death. A bequest motive can then be proxied by lowering δ_2. To gauge the impact of this on $\Psi^y(1)$ (i.e., the effect that a permanent increase in real rates has on consumption demand) the left panel of Figure 5 regenerates our heatmap after reducing δ_2 from $\frac{1}{20}$ to

$\frac{1}{30}$. As the figure shows, the range of parameter values where $\Psi^y(1)$ is close to zero (or positive) expands – centered around slightly higher values of the EIS. For example, with an EIS just below 0.3, there is now a large range for μ (governing asset duration) where $\Psi^y(1)$ is small.

Equity. Agents only hold government bonds in our model. This may seem restrictive, as it neglects equity. Introducing an equity market is straightforward. In our current setup, working households own all firms. Alternatively, firm equity could be traded in a market featuring both workers and retirees, with the equity price responding to interest rates as implied by standard arbitrage conditions. We have explored this modification and have not found it to affect our main results – motivating our choice for the simpler setup. The reason for this robustness lies in the fact that interest rates affect equity and long-term bond prices in the same direction. So, while the introduction of equity would make the model's asset valuation channel more involved, it would not change its nature. There are nonetheless two aspects that would change with equity. The first concerns the strength of the valuation channel. With only long-term bonds, the strength of this channel is governed by bond duration. But with equity, the strength would also be governed by the equity risk premium.[36] This does not change the main mechanism, but influences how to calibrate the model (as discussed in footnote 30). The second aspect that would change with equity, is that it would open the door to exploring changes in risk premiums (Caramp and Silva (2021) offer an analysis along these lines), which is also related to the literature on safe asset demand (Caballero et al., 2016; 2017). At the moment, the mechanism via which the central bank in our model can affect longer-term rates runs entirely through the expectations hypothesis. But a "low for long" policy might also trigger a search for yield, making investors move more into equity. We suspect that such a shift would compress the equity premium, making the risky rate of return fall by more than the safe rate – *adding* to the retirement savings challenge – but leave a formal analysis for future work.[37]

Housing. Along similar lines, the logic of the model would continue to hold if households were allowed to save in a housing asset. So, while our model contains a long-term bond as

[36]The steady-state value of equity would equal $\frac{d}{r+rp}$, where d is the dividend payment, r is the real rate, and rp is an equity risk premium. Recall that the steady-state bond price in the model is given by $\frac{1}{r+\mu}$, where $1/\mu$ governs bond duration. This illustrates that a lower equity premium implies that asset prices are more sensitive to real rate changes, which parallels the role played by bond duration.

[37]In reality, the risk premium appears to have *risen* since the 1980s (for reasons outside of our model), implying that the risky rate of return has fallen by less than the safe rate (Reis, 2022). This is no panacea for retirement savings though, as investing in riskier options implies that one ends up with a different (riskier) pension, to which one might again respond by wishing to hold more assets.

the asset through which saving takes place, the exact nature of the asset is of secondary importance. The more important issue is that this asset has positive duration, i.e., that its price "q" is inversely related to the interest rate.

Physical capital. A next extension in this line is to enrich the model with productive physical capital K. While one might think that the accompanying "investment channel" of monetary policy could overturn some of our findings, this turns out not to be so. This is shown by the right panel of Figure 5, which plots the heatmap after extending our model with capital and quadratic investment adjustment costs (details are in Appendix E). It looks similar to Figure 4, which abstracts from capital, suggesting that is a decent approximation for the question central to this paper. To understand why, it helps to think of r^* as the interest rate which sets long-run excess demand ("XD") to 0. The natural logarithm of this object can be defined as $\ln XD = \ln(C + I) - \ln F(K, L) = \ln(C + \nu K) - \ln F(K, L)$, where "$\nu$" is capital's depreciation rate. Differentiating with respect to $\ln r$, whilst holding consumption and labor supply constant, gives $\frac{\partial \ln XD}{\partial \ln r}\Big|_{C,L} = \left(\frac{I}{Y} - \frac{K \cdot \partial F/\partial K}{F(K,L)}\right) \frac{\partial \ln K}{\partial \ln r}$. Since the investment share $\frac{I}{Y}$ tends to be smaller than the capital share $\frac{K \cdot \partial F/\partial K}{F(K,L)}$ (in the U.S., the former is about 20% versus 30% for the latter), one can see that the partial effect of higher interest rates is to create excess *demand* (not excess supply) in the natural case where $\frac{\partial \ln K}{\partial \ln r} < 0$. The reason lies in the fact that investment does not just come with a demand aspect to it, but also affects future supply; on balance, investment tends to expand long-run supply by more than long-run investment demand in realistic calibrations. Hence, what is required for excess demand to be strongly negatively related to r, is that consumption is strongly negatively related to r. This explains our focus on the latter.

Local versus global. While we only offer a local analysis of our model in this paper, it is relevant to mention how results would change with a global analysis. In our local analysis, real rates can deviate from r^* for long periods of time without doing much to activity or inflation. However, if the deviation became very large, many of the local properties could change. As shown in Beaudry et al. (2024), the underlying asset demand function is C-shaped. This means that, at very high real rates, asset demand will eventually always become increasing in returns (even for $EIS \ll 1$). This implies that large deviations in interest rates away from r^* would not be possible without creating a large economic boom or contraction. Hence, from a global perspective, r^* should be viewed as remaining relevant, but knowing it with great precision might not be very important.

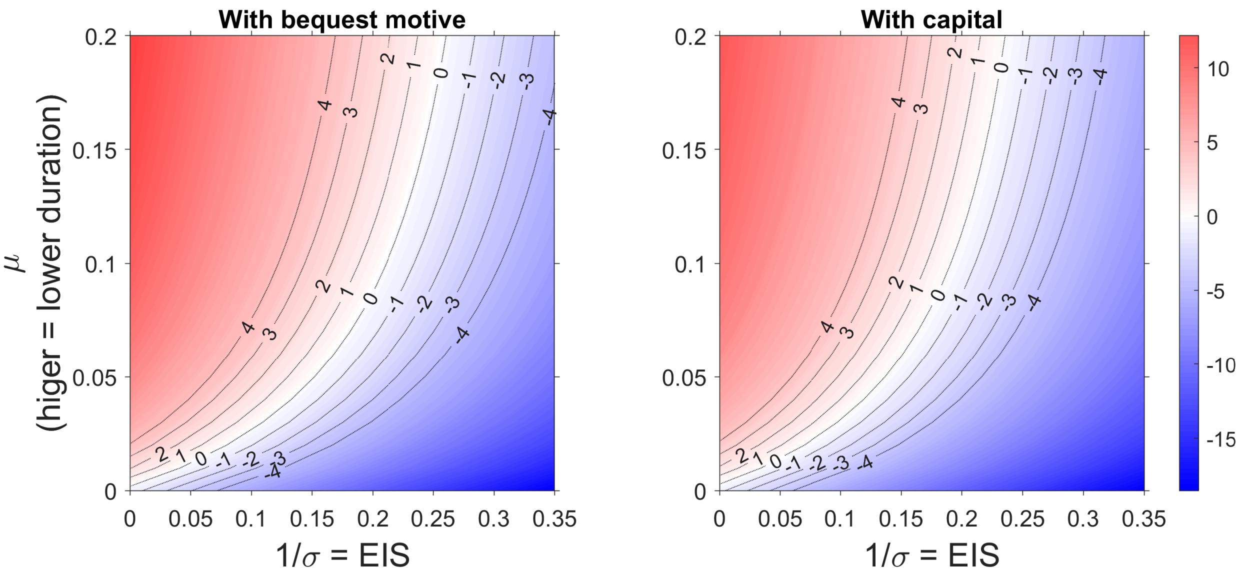

Figure 5: $\Psi^y(1)$ as a function of σ and μ in FLANK. The left panel shows this when proxying a bequest motive by setting $\delta_2 = 1/30$, while the right panel extends the model with capital. Other parameters are calibrated as in footnote 23.

6.2 Possible extensions for future work

By offering a highly tractable framework combining life-cycle forces and monetary policy, our work opens several avenues for future work. Our finding that conventional monetary policy may be less potent when retirement preoccupations are more prevalent (or when household assets are of shorter duration) suggests that central banks may need to move the interest rate *by more* to achieve a given effect on output and prices in an aging society (or a "post-QE world" where central banks hold significant long-term bond portfolios). This may have adverse consequences for financial stability. We do not model these interactions here, but such an extension could be warranted.

Second, while FLANK is already heterogenous-agent in nature (distinguishing between workers and retirees), it could be interesting to incorporate other dimensions of heterogeneity. A natural candidate would involve heterogeneity in the MPC out of wealth. Empirical studies document that this object varies across the wealth distribution, with richer households having lower MPCs (Di Maggio et al., 2020; Chodorow-Reich et al., 2021). In that case, our model's logic suggests that greater inequality (a smaller fraction of households owning a bigger share of the asset supply) can weaken the monetary transmission mechanism – as the "asset valuation effect" is normally an important force working in the conventional direction. But when consumption demand of asset holders is not very sensitive to valuation effects, as would be the case when most assets are held by low-MPC households, this channel loses potency. To analyze such questions, the model developed by Bardoczy and Velasquez-Giraldo (2024), which combines MPC-heterogeneity with life-cycle dynamics, seems to hold great potential.

When it comes to adding realism, countries typically do not solely rely on funded pension arrangements – also providing retirees with a basic retirement income via a pay-as-you-go (PAYG) system, financed by taxing workers. The generosity of such schemes however tends to be limited,[38] leaving an important role for the saving dynamics central to our paper – a role that would only gain importance if one were to explicitly model bequest motives (in contrast to savings, a PAYG pension cannot be bequeathed to one's offspring). What our model also makes clear, is that the importance of retirement preoccupations to the monetary transmission mechanism is greater when PAYG pensions are less generous. As demographic forces (higher old-age dependency ratios) are putting PAYG systems under pressure (OECD, 2021), our paper suggests that the importance of retirement preoccupations to monetary policy may rise further over time.

Our model can also serve as a guide to empirical researchers in formulating the correct specification when trying to estimate the MPC out of wealth – with our model showing how and why to control for the accompanying *level* of interest rates. If wealth levels are high because of low discount rates, the MPC is likely to be low, as households would want to hold on to their assets to compensate for the lower flow return. This suggests that the MPC out of financial wealth not only varies with wealth holdings (with richer households having a lower MPC) but also with the prevailing level of long-term rates. Recent empirical findings in Di Maggio et al. (2020) and Fagereng et al. (2021) are indeed hinting in this direction, pointing towards a higher MPC out of dividend payouts relative to capital gains stemming from lower rates of interest.

It would also be interesting to characterize optimal policy in FLANK. Since the model suggests that very persistent rate changes might not affect demand by much, this implies that interest rate policy may be ill-equipped to offset persistent demand shocks. The latter may be better left to fiscal policy, with monetary policy instead focusing on stabilization in response to disturbances that are deemed more transient in nature.

Finally, to us, the region of the model's parameter space where $\Psi^y(1) \approx 0$ carries considerable appeal: not only can it explain why central banks appear to have significant control over longer-term real rates, but also why central banks have been quite successful

[38]For example: 2023 U.S. Social Security payments were about $1,782 per month (see https://www.cbpp.org/sites/default/files/atoms/files/8-8-16socsec.pdf). Most young, working Americans are moreover pessimistic about their future Social Security benefits (Turner and Rajnes, 2021), increasing the importance of their own saving efforts.

in fulfilling their mandate despite being very imperfectly informed about the location of r^*. In this light, it is interesting to explore what can widen the range where $\Psi^y(1)$ is small. Our initial explorations suggest that a bequest motive can do so (recall the discussion around Figure 5) but there may be other avenues. One possibility is to consider Epstein-Zin (1989) preferences, which allow the EIS and coefficient of relative risk aversion to be calibrated separately (rather than imposing that they are each other's inverse).

7 Conclusion

There is considerable evidence suggesting central banks' policy rate decisions have a significant effect on long-term real rates. A common interpretation is that this reflects reverse causality – with central banks having significant private information about the value of r^* and this information being transmitted to markets around policy decisions.

In this paper we instead argue that this link may actually have a causal element to it, albeit not deliberate. We developed a New Keynesian-type model with life-cycle features ("FLANK") to highlight the potential effects of very persistent policy-induced changes in interest rates. In this setup, we show that very persistent rate changes involve different effects (rooted in intertemporal substitution, asset valuation, and asset demand) that act on aggregate demand in opposing directions and that together imply an ambiguous effect on economic activity. Standard calibrations suggests that the net effect of very persistent policy-induced rate changes may be close to zero.

While we do not claim to know with certainty that the net effects are in fact approximately zero – even though it is consistent with various empirical observations and calibrations offered in this paper – we do argue that such a possibility opens the door to a fundamentally different view regarding the powers of central banks. Especially, it offers an interpretation on the observed link between policy rates and long-term real rates that does not rely on central banks having private information. According to our perspective, there is a "persistence-potency trade-off" and the persistent component of monetary policy is much less potent than commonly thought. Our FLANK model implies that if a central bank chooses to keep real rates low for a prolonged period, as many central banks did post-GFC, this may not boost the economy much; it might even cause a slight contraction. The main effect of such a low-for-long policy would be to lower long-term rates and boost asset valuations. But that might not stimulate consumption demand as households choose to hold on to this expanded wealth, given it is now expected to generate less flow income going forward (implying that the household is not able to afford more life-time consumption).

As a result, if central banks misperceive r^*, and used their misperceived r^* to guide policy, they would have very few signals suggesting they are mistaken. In this sense, the economy is rather forgiving to a central bank community that misperceives r^*. Accordingly, central bank decisions may actually drive real rates over long periods of time, without them realizing this. It can lead to cases where a rate cut that the central bank initially intends to be purely temporary, acquires additional persistence as it subsequently induces the central bank to erroneously lower its estimate of r^* (and vice versa for a rate hike). In this type of environment, it becomes rational for markets to view central bank decisions and statements as relevant for long-term rates, even if they do not think central banks have private information about r^*.

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Appendix

A Equilibrium and steady state

The equilibrium of the model is described by the following equations:

$$
\begin{aligned}
y_t &= \frac{\vartheta c_t^w + (1-\vartheta)\, c_t^r}{1 - \frac{\theta}{2}\left(\pi_t - \bar{\pi}\right)^2} \\
c_t^r &= a_t^r \left[(\Gamma_t)^{-1} - 1 \right]^{-1} \\
\left(c_t^w\right)^{-\sigma} &= \beta_t \left\{ (1-\delta_1)\, \mathbb{E}_t \left[\left(c_{t+1}^w\right)^{-\sigma} r_{t+1} \right] + \delta_1 \mathbb{E}_t \left[\left(a_t^w r_{t+1}^w + \tau_{t+1}^r \right)^{-\sigma} (\Gamma_{t+1})^{-\sigma} r_{t+1} \right] \right\} \\
\left[(\Gamma_t)^{-1} - 1 \right]^{\sigma} &= (1-\delta_2)\, \beta_t \mathbb{E}_t \left[r_{t+1} \left(r_{t+1}^r \Gamma_{t+1} \right)^{-\sigma} \right] \\
(\pi_t - \bar{\pi})\, \pi_t &= \lambda \left[\left(\frac{y_t}{\vartheta A} \right)^{1+\varphi} - 1 \right] + \mathbb{E}_t \left[\Lambda_{t,t+1}^w \left(\pi_{t+1} - \bar{\pi} \right) \pi_{t+1} \frac{y_{t+1}}{y_t} \right] \\
\Lambda_{t,t+1}^w &= \beta_t \frac{(1-\delta_1) \left(c_{t+1}^w\right)^{-\sigma} + \delta_1 \left(a_t^w r_{t+1}^w + \tau_{t+1}^r \right)^{-\sigma} (\Gamma_{t+1})^{-\sigma}}{\left(c_t^w\right)^{-\sigma}} \\
\Lambda_{t,t+1}^r &= (1-\delta_2)\, \beta \frac{\left(r_{t+1}^r \Gamma_{t+1} \right)^{-\sigma}}{\left(\Gamma_t^{-1} - 1 \right)^{\sigma}} \\
q_t b^g &= \vartheta a_t^w + (1-\vartheta)\, a_t^r \\
0 &= \vartheta \left(1 - \alpha_t^w\right) a_t^w + (1-\vartheta) \left(1 - \alpha_t^r\right) a_t^r \\
r_{t+1}^r &= r_{t+1} + \left[\frac{1 + (1-\mu)\, q_{t+1}}{q_t} - r_{t+1} \right] \alpha_t^r \\
r_{t+1}^w &= r_{t+1} + \left[\frac{1 + (1-\mu)\, q_{t+1}}{q_t} - r_{t+1} \right] \alpha_t^w \\
1 &= \mathbb{E}_t \left[\Lambda_{t,t+1}^r \frac{1 + (1-\mu)\, q_{t+1}}{q_t} \right] \\
1 &= \mathbb{E}_t \left[\Lambda_{t,t+1}^w \frac{1 + (1-\mu)\, q_{t+1}}{q_t} \right] \\
a_t^r &= \left[(1-\delta_2)\, a_{t-1}^r r_t^r + \delta_2 \left(a_{t-1}^w r_t^w + \tau_t^r \right) \right] (1 - \Gamma_t) \\
i_t &= r \bar{\pi} \left(\frac{\mathbb{E}_t \left[\pi_{t+1} \right]}{\bar{\pi}} \right)^{1+\phi} e^{\varepsilon_t^i} \\
r_{t+1} &= \frac{i_t}{\pi_{t+1}}
\end{aligned}
$$

For a zero inflation target ($\bar{\pi} = 1$) and $\tau^r = 0$, the steady-state real rate r solves:

$$\frac{y}{r-\left[\left(1-\delta_2\right)\beta r\right]^{\frac{1}{\sigma}}}\frac{1+\delta_1\frac{\left[(1-\delta_2)\beta r\right]^{\frac{1}{\sigma}}}{1-(1-\delta_2)\left[(1-\delta_2)\beta r\right]^{\frac{1}{\sigma}}}}{\left[\frac{1-(1-\delta_1)\beta r}{\delta_1\beta r}\right]^{\frac{1}{\sigma}}+\frac{\delta_1}{1-(1-\delta_2)\left[(1-\delta_2)\beta r\right]^{\frac{1}{\sigma}}}}=\frac{b^g}{r-1+\mu}$$

The left-hand side of this equation represents the steady-state demand for savings, while the right-hand side captures the steady-state value of the assets supplied to the economy. Let $\gamma\equiv\left(1-\vartheta\right)c^r/y$ denote the share of steady-state output consumed by retirees, and $\varsigma\equiv a^r/a^w$ denote the steady-state financial wealth of retirees relative to workers. Their equations are

$$\begin{aligned}
\varsigma &= \frac{\delta_2\left[\left(1-\delta_2\right)\beta r\right]^{\frac{1}{\sigma}}}{1-\left(1-\delta_2\right)\left[\left(1-\delta_2\right)\beta r\right]^{\frac{1}{\sigma}}}\\
\gamma &= \frac{\delta_1}{\left[\frac{1-(1-\delta_1)\beta r}{\delta_1\beta r}\right]^{\frac{1}{\sigma}}\left\{1-\left(1-\delta_2\right)\left[\left(1-\delta_2\right)\beta r\right]^{\frac{1}{\sigma}}\right\}+\delta_1}
\end{aligned}$$

Now assume that $a_t^r=\varsigma a_t^w$, $r=\beta^{-1}$ and τ_{t+1}^r is unexpected. The log-linearized equilibrium equations are then given by:

$$\begin{aligned}
\hat{y}_t &= \left(1-\gamma\right)\hat{c}_t^w+\gamma\hat{c}_t^r\\
\hat{c}_t^r &= \hat{q}_t+\left[\beta\left(1-\delta_2\right)^{\frac{1}{\sigma}}\right]^{-1}\hat{\Gamma}_t\\
\hat{c}_t^w &= \left(1-\delta_1\right)\left(\mathbb{E}_t\hat{c}_{t+1}^w-\frac{1}{\sigma}\mathbb{E}_t\hat{r}_{t+1}\right)+\delta_1\left(\hat{q}_t+\left[\beta\left(1-\delta_2\right)^{\frac{1}{\sigma}}\right]^{-1}\hat{\Gamma}_t\right)-\frac{1-\delta_1}{\sigma}\varepsilon_t^\beta\\
\hat{\Gamma}_t &= \beta\left(1-\delta_2\right)^{\frac{1}{\sigma}}\left[\mathbb{E}_t\hat{\Gamma}_{t+1}+\frac{\sigma-1}{\sigma}\mathbb{E}_t\hat{r}_{t+1}-\frac{1}{\sigma}\varepsilon_t^\beta\right]\\
\hat{\pi}_t &= \lambda\left(1+\varphi\right)\hat{y}_t+\beta\mathbb{E}_t\hat{\pi}_{t+1}\\
\hat{q}_t &= \beta\left(1-\mu\right)\mathbb{E}_t\hat{q}_{t+1}-\mathbb{E}_t\hat{r}_{t+1}\\
\hat{r}_{t+1} &= \hat{i}_t-\mathbb{E}_t\hat{\pi}_{t+1}-\varrho\\
\hat{i}_t &= \varrho+\left(1+\phi\right)\mathbb{E}_t\hat{\pi}_{t+1}+\varepsilon_t^i
\end{aligned}$$

with $a_t^w=a_t^r=q_t$, $r_{t+1}^r=r_{t+1}^w=r_{t+1}$, and $\varrho\equiv\log r$.

B Proofs of Propositions

B.1 Proof of Proposition 1

When $\phi=0$, the equilibrium dynamics are captured by:

$$\begin{bmatrix} \hat{c}^w_t \\ \hat{\Gamma}_t \\ \hat{\pi}_t \\ \hat{q}_t \end{bmatrix} = \begin{bmatrix} 1-\delta_1 & \delta_1 & 0 & \beta\delta_1(1-\mu) \\ 0 & \beta(1-\delta_2)^{\frac{1}{\sigma}} & 0 & 0 \\ \kappa(1-\gamma)(1-\delta_1) & \kappa(1-\gamma)\delta_1+\kappa\gamma & \beta & \kappa\beta(1-\mu)[(1-\gamma)\delta_1+\gamma] \\ 0 & 0 & 0 & \beta(1-\mu) \end{bmatrix} \begin{bmatrix} \mathbb{E}_t\hat{c}^w_{t+1} \\ \mathbb{E}_t\hat{\Gamma}_{t+1} \\ \mathbb{E}_t\hat{\pi}_{t+1} \\ \mathbb{E}_t\hat{q}_{t+1} \end{bmatrix}$$

The four eigenvalues of this system are $\left\{\beta, \beta(1-\mu), 1-\delta_1, \beta(1-\delta_2)^{1/\sigma}\right\}$. Since $\beta, \mu, \delta_1, \delta_2 \in (0,1)$ and $\sigma > 0$ then all four eigenvalues are less than 1 in modulus and the system has a unique stable solution.

B.2 Proof of Proposition 2

We start by deriving the "yield curve representation" of $\hat{y}_t$ and $\hat{\pi}_t$, equations (21) and (22). Assume $\phi = 0$, such that $\hat{r}_{t+1} = \varepsilon^i_t$. Solving q and Γ forward yields

$$\hat{q}_t = -\mathbb{E}_t\hat{r}_{t+1} - \sum_{j=1}^{\infty}\beta^j(1-\mu)^j\,\mathbb{E}_t\hat{r}_{t+1+j}$$

$$\hat{\Gamma}_t = \frac{\sigma-1}{\sigma}\sum_{j=0}^{\infty}\left[\beta(1-\delta_2)^{\frac{1}{\sigma}}\right]^{j+1}\mathbb{E}_t\hat{r}_{t+1+j} - \frac{1}{\sigma}\sum_{j=0}^{\infty}\left[\beta(1-\delta_2)^{\frac{1}{\sigma}}\right]^{j+1}\varepsilon^{\beta}_{t+j}$$

Plug these into the workers' Euler equation to obtain

$$\hat{c}^w_t = (1-\delta_1)\,\mathbb{E}_t\hat{c}^w_{t+1} - \frac{1}{\sigma}\mathbb{E}_t r_{t+1} + \delta_1\sum_{j=1}^{\infty}\beta^j\left[\frac{\sigma-1}{\sigma}(1-\delta_2)^{\frac{j}{\sigma}} - (1-\mu)^j\right]\mathbb{E}_t r_{t+1+j}$$
$$-\frac{1}{\sigma}\left[\delta_1\sum_{j=1}^{\infty}\beta^j(1-\delta_2)^{\frac{j}{\sigma}}\,\mathbb{E}_t\varepsilon^{\beta}_{t+j} + \varepsilon^{\beta}_t\right]$$

Through repeated substitution, we can express $\hat{c}^w_t$ as a function of future real interest rates and demand shocks only, as follows:

$$\hat{c}^w_t = \sum_{j=0}^{\infty}\tilde{\psi}_j\mathbb{E}_t\hat{r}_{t+1+j} + \sum_{j=0}^{\infty}\tilde{\omega}_j\mathbb{E}_t\varepsilon^{\beta}_{t+j}$$

where $\tilde{\psi}_0 = \tilde{\omega}_0 = -\frac{1}{\sigma}$ and

$$\tilde{\psi}_j = \tilde{\psi}_{j-1}(1-\delta_1) + \delta_1\beta^j\left[\frac{\sigma-1}{\sigma}(1-\delta_2)^{\frac{j}{\sigma}} - (1-\mu)^j\right]$$
$$= (1-\delta_1)^j\,\tilde{\psi}_0 + \delta_1\sum_{i=1}^{j}(1-\delta_1)^{j-i}\beta^i\left[\frac{\sigma-1}{\sigma}(1-\delta_2)^{\frac{i}{\sigma}} - (1-\mu)^i\right]$$

$$
\begin{aligned}
\tilde{\omega}_j &= \tilde{\omega}_{j-1}(1-\delta_1) - \frac{\delta_1}{\sigma}\beta^j(1-\delta_2)^{\frac{j}{\sigma}} \\
&= (1-\delta_1)^j\tilde{\omega}_0 - \frac{\delta_1}{\sigma}\sum_{i=1}^{j}(1-\delta_1)^{j-i}\beta^i(1-\delta_2)^{\frac{i}{\sigma}}
\end{aligned}
$$

Now, plug the equations derived above for q and Γ into the retirees' consumption function to obtain a similar representation for $\hat{c}_t^r$:

$$
\hat{c}_t^r = \sum_{j=0}^{\infty}\bar{\psi}_j\mathbb{E}_t\hat{r}_{t+1+j} + \sum_{j=0}^{\infty}\bar{\omega}_j\mathbb{E}_t\varepsilon_{t+j}^{\beta}
$$

where $\bar{\psi}_0 = \bar{\omega}_0 = -\frac{1}{\sigma}$ and

$$
\begin{aligned}
\bar{\psi}_j &= \beta^j\left[\frac{\sigma-1}{\sigma}(1-\delta_2)^{\frac{j}{\sigma}} - (1-\mu)^j\right] \\
\bar{\omega}_j &= -\frac{1}{\sigma}\beta^j(1-\delta_2)^{\frac{j}{\sigma}}
\end{aligned}
$$

Finally, we can use these representations for $\hat{c}_t^w$ and $\hat{c}_t^r$ to rewrite $\hat{y}_t$ as

$$
\hat{y}_t = \sum_{j=0}^{\infty}\psi_j^y\mathbb{E}_t\hat{r}_{t+1+j} + \sum_{j=0}^{\infty}\omega_j^y\mathbb{E}_t\varepsilon_{t+j}^{\beta}
$$

where $\psi_j^y \equiv (1-\gamma)\tilde{\psi}_j + \gamma\bar{\psi}_j$ and $\omega_j^y \equiv (1-\gamma)\tilde{\omega}_j + \gamma\bar{\omega}_j$, which imply $\psi_0^y = \omega_0^y = -\frac{1}{\sigma}$ and

$$
\begin{aligned}
\psi_j^y &= -\frac{1}{\sigma}(1-\gamma)(1-\delta_1)^j + (1-\gamma)\delta_1\sum_{i=1}^{j}(1-\delta_1)^{j-i}\beta^i\left[\frac{\sigma-1}{\sigma}(1-\delta_2)^{\frac{i}{\sigma}} - (1-\mu)^i\right] \\
&\quad + \gamma\beta^j\left[\frac{\sigma-1}{\sigma}(1-\delta_2)^{\frac{j}{\sigma}} - (1-\mu)^j\right] \\
&= (1-\delta_1)\psi_{j-1}^y + \frac{\sigma-1}{\sigma}\left[\delta_1 - \gamma(1-\delta_1)\frac{1-\beta(1-\delta_2)^{\frac{1}{\sigma}}}{\beta(1-\delta_2)^{\frac{1}{\sigma}}}\right]\beta^j(1-\delta_2)^{\frac{j}{\sigma}} \\
&\quad - \left[\delta_1 - \gamma(1-\delta_1)\frac{1-\beta(1-\mu)}{\beta(1-\mu)}\right]\beta^j(1-\mu)^j \\
\omega_j^y &= -\frac{1}{\sigma}(1-\gamma)(1-\delta_1)^j + \frac{1}{\sigma}(1-\gamma)\delta_1\sum_{i=1}^{j}(1-\delta_1)^{j-i}\beta^i(1-\delta_2)^{\frac{i}{\sigma}} - \gamma\frac{1}{\sigma}\beta^j(1-\delta_2)^{\frac{j}{\sigma}} \\
&= (1-\delta_1)\omega_{j-1}^y - \frac{1}{\sigma}\left[\delta_1 - \gamma(1-\delta_1)\frac{1-\beta(1-\delta_2)^{\frac{1}{\sigma}}}{\beta(1-\delta_2)^{\frac{1}{\sigma}}}\right]\beta^j(1-\delta_2)^{\frac{j}{\sigma}}
\end{aligned}
$$

Finally, solve $\hat{\pi}_t$ forward to obtain $\hat{\pi}_t = \kappa\sum_{j=0}^{\infty}\beta^j\mathbb{E}_t\hat{y}_{t+j}$. Then use the representation derived above to express $\hat{\pi}_t$as follows:

$$\hat{\pi}_t = \sum_{j=0}^{\infty} \psi_j^{\pi} \mathbb{E}_t \hat{r}_{t+1+j} + \sum_{j=0}^{\infty} \omega_j^{\pi} \mathbb{E}_t \varepsilon_{t+j}^{\beta}$$

where $\psi_0^{\pi} = \kappa \psi_0^{y}$, $\omega_0^{\pi} = \kappa \omega_0^{y}$, and

$$\psi_j^{\pi} = \beta \psi_{j-1}^{\pi} + \kappa \psi_j^{y}$$
$$\omega_j^{\pi} = \beta \omega_{j-1}^{\pi} + \kappa \omega_j^{y}$$

Proof of 2.a If $\delta_1 = 0$, then $\psi_j^y = -\frac{1}{\sigma}$ and $\psi_j^{\pi} = -\frac{\kappa}{\sigma} \frac{1-\beta^{j+1}}{1-\beta}$, for all $j \geq 0$. If $\delta_1 > 0$, then

$$\psi_1^y = -\frac{1}{\sigma} + \frac{1}{\sigma} \left[\delta_1 + \gamma (1-\delta_1)\right] \left[1 - \beta (1-\delta_2)^{\frac{1}{\sigma}}\right] + \left[\delta_1 + \gamma (1-\delta_1)\right] \left[\beta (1-\delta_2)^{\frac{1}{\sigma}} - \beta (1-\mu)\right]$$

$$\psi_2^y = -\frac{1}{\sigma} + \frac{1}{\sigma} \left\{ \left[1 - \delta_1 + \beta (1-\delta_2)^{\frac{1}{\sigma}}\right] \left[\delta_1 + \gamma (1-\delta_1)\right] + \delta_1 \right\} \left[1 - \beta (1-\delta_2)^{\frac{1}{\sigma}}\right]$$
$$+ \left\{ \left[\delta_1 + \gamma (1-\delta_1)\right] \left[\beta (1-\delta_2)^{\frac{1}{\sigma}} + \beta (1-\mu)\right] + \delta_1 (1-\gamma)(1-\delta_1) \right\} \left[\beta (1-\delta_2)^{\frac{1}{\sigma}} - \beta (1-\mu)\right]$$

$$\psi_3^y = ...$$

If $\delta_2 < \mu$, then they are all strictly grater than $-\frac{1}{\sigma}$. Since $\psi_j^y > -\frac{1}{\sigma}$ for all $j \geq 1$ and $\psi_j^{\pi} = \kappa \sum_{i=0}^{j} \beta^{j-i} \psi_i^y$, then also $\psi_j^{\pi} > -\frac{\kappa}{\sigma} \frac{1-\beta^{j+1}}{1-\beta}$ for all $j \geq 1$.

Proof of 2.b Solve ψ_j^y backward to express it as follows:

$$\psi_j^y = (1-\delta_1)^j \psi_0^y + \sum_{i=1}^{j} (1-\delta_1)^{j-i} \xi_i^{\psi}$$

where $\xi_i^{\psi} \equiv \frac{\sigma-1}{\sigma} \left[\delta_1 - \frac{\gamma}{(1-\delta_1)^{-1}} \frac{1-\beta(1-\delta_2)^{\frac{1}{\sigma}}}{\beta(1-\delta_2)^{\frac{1}{\sigma}}}\right] \beta^j (1-\delta_2)^{\frac{i}{\sigma}} - \left[\delta_1 - \frac{\gamma}{(1-\delta_1)^{-1}} \frac{1-\beta(1-\mu)}{\beta(1-\mu)}\right] \beta^i (1-\mu)^i$. Now, since $\lim_{i\to\infty} \xi_i^{\psi} = 0$ then also $\lim_{j\to\infty} \psi_j^y = 0$, provided that $\delta_1 > 0$. Since $\psi_j^{\pi} = \kappa \sum_{i=0}^{j} \beta^{j-i} \psi_i^y$, then also $\lim_{j\to\infty} \psi_j^{\pi} = 0$.

Proof of 2.c The derivative of ψ_j^y with respect to σ is

$$\frac{\partial \psi_j^y}{\partial \sigma} = \frac{1}{\sigma^2} (1-\gamma)(1-\delta_1)^j + \gamma \beta^j \left[\frac{1}{\sigma^2} (1-\delta_2)^{\frac{j}{\sigma}} + \frac{\sigma-1}{\sigma} (1-\delta_2)^{\frac{j}{\sigma}} \left[-\ln(1-\delta_2)\right] \frac{j}{\sigma^2}\right]$$
$$+ (1-\gamma) \delta_1 \sum_{i=1}^{j} (1-\delta_1)^{j-i} \beta^i \left[\frac{1}{\sigma^2} (1-\delta_2)^{\frac{i}{\sigma}} + \frac{\sigma-1}{\sigma} (1-\delta_2)^{\frac{i}{\sigma}} \left[-\ln(1-\delta_2)\right] \frac{i}{\sigma^2}\right]$$

Since all of the terms are positive (recall that $\delta_2 \in [0,1]$, therefore $-\ln(1-\delta_2) > 0$), then $\partial \psi_j^y / \partial \sigma > 0$. The derivative of ψ_j^{π} with respect to σ is $\partial \psi_j^{\pi} / \partial \sigma = \kappa \sum_{i=0}^{j} \beta^{j-i} \partial \psi_i^y / \partial \sigma$, which is therefore also positive. Finally, note that

$$\lim_{\sigma\to+\infty}\psi_j^y=(1-\gamma)\,\delta_1\sum_{i=1}^{j}(1-\delta_1)^{j-i}\,\beta^i\left[1-(1-\mu)^i\right]+\gamma\beta^j\left[1-(1-\mu)^j\right]>0$$

which is strictly positive, as $\mu\in(0,1]$. Since ψ_j^y is continuous in σ and its limit for $\sigma\to+\infty$ is positive, then $\exists\sigma<+\infty$ such that $\psi_j^y>0$. Similarly, since $\psi_j^\pi=\kappa\sum_{i=0}^{j}\beta^{j-i}\psi_i^y$ is continuous in σ and $\lim_{\sigma\to+\infty}\psi_j^\pi=\kappa\sum_{i=0}^{j}\beta^{j-i}\lim_{\sigma\to+\infty}\psi_i^y>0$, then $\exists\sigma<+\infty$ such that $\psi_j^\pi>0$.

Proof of 2.d The derivatives of ψ_j^y with respect to δ_2 and μ are

$$\begin{aligned}\frac{\partial\psi_j^y}{\partial\delta_2}&=-\frac{1}{\sigma}\frac{\sigma-1}{\sigma}\frac{(1-\gamma)\,\delta_1\sum_{i=1}^{j}(1-\delta_1)^{j-i}\,\beta^i i\,(1-\delta_2)^{\frac{i}{\sigma}}+\gamma\beta^j j\,(1-\delta_2)^{\frac{j}{\sigma}}}{1-\delta_2}<0,\\ \frac{\partial\psi_j^y}{\partial\mu}&=\frac{(1-\gamma)\,\delta_1\sum_{i=1}^{j}(1-\delta_1)^{j-i}\,\beta^i i\,(1-\mu)^i+\gamma\beta^j j\,(1-\mu)^j}{1-\mu}>0.\end{aligned}$$

Since $\psi_j^\pi=\kappa\sum_{i=0}^{j}\beta^{j-i}\psi_i^y$, the derivatives of ψ_j^π with respect to δ_2 and μ are

$$\begin{aligned}\frac{\partial\psi_j^\pi}{\partial\delta_2}&=\kappa\sum_{i=0}^{j}\beta^{j-i}\frac{\partial\psi_i^y}{\partial\delta_2}<0,\\ \frac{\partial\psi_j^\pi}{\partial\mu}&=\kappa\sum_{i=0}^{j}\beta^{j-i}\frac{\partial\psi_i^y}{\partial\mu}>0.\end{aligned}$$

B.3 Proof of Proposition 3

We start by deriving equation (25). Assume $\phi=0$ and $\mathbb{E}_t\varepsilon_{t+1}^i=\rho_i\varepsilon_t^i$. Then $\hat{y}_t=\sum_{j=0}^{\infty}\psi_j^y\mathbb{E}_t\hat{r}_{t+1+j}=\sum_{j=0}^{\infty}\psi_j^y\,(\rho_i)^j\,\varepsilon_t^i=\Psi^y\,(\rho_i)\,\varepsilon_t^i$, where

$$\begin{aligned}\Psi^y(\rho_i)&=-\frac{1}{\sigma}+\sum_{j=1}^{\infty}(1-\delta_1)\psi_{j-1}^r\rho_i^j-\delta_1\left[1-\gamma\frac{1-\delta_1}{\delta_1}\frac{1-\beta(1-\mu)}{\beta(1-\mu)}\right]\sum_{j=1}^{\infty}(\beta\rho_i)^j(1-\mu)^j\\ &\quad+\delta_1\frac{\sigma-1}{\sigma}\left[1-\gamma\frac{1-\delta_1}{\delta_1}\frac{1-\beta(1-\delta_2)^{\frac{1}{\sigma}}}{\beta(1-\delta_2)^{\frac{1}{\sigma}}}\right]\sum_{j=1}^{\infty}(\beta\rho_i)^j(1-\delta_2)^{\frac{j}{\sigma}}\\ &=-\frac{1}{\sigma}+(1-\delta_1)\rho_i\Psi(\rho_i)-\delta_1\left[1-\gamma\frac{1-\delta_1}{\delta_1}\frac{1-\beta(1-\mu)}{\beta(1-\mu)}\right]\frac{\beta\rho_i(1-\mu)}{1-\beta\rho_i(1-\mu)}\\ &\quad+\delta_1\frac{\sigma-1}{\sigma}\left[1-\gamma\frac{1-\delta_1}{\delta_1}\frac{1-\beta(1-\delta_2)^{\frac{1}{\sigma}}}{\beta(1-\delta_2)^{\frac{1}{\sigma}}}\right]\frac{\beta\rho_i(1-\delta_2)^{\frac{1}{\sigma}}}{1-\beta\rho_i(1-\delta_2)^{\frac{1}{\sigma}}}\\ &=-\frac{1}{\sigma}\frac{(1-\gamma)(1-\delta_1)}{1-\rho_i(1-\delta_1)}+\left[\gamma+\frac{\delta_1(1-\gamma)}{1-\rho_i(1-\delta_1)}\right]\left[\frac{\sigma-1}{\sigma}\frac{1}{1-\beta\rho_i(1-\delta_2)^{\frac{1}{\sigma}}}-\frac{1}{1-\beta\rho_i(1-\mu)}\right]\end{aligned}$$

Now, if $\delta_1 = 0$ then $\Psi^y(\rho_i) = -\frac{1}{\sigma}\frac{1}{1-\rho_i}$. This expression is strictly negative, for all $\rho_i \in [0,1)$, and diverges to $-\infty$ as $\rho_i \uparrow 1$.

B.4 Proof of Proposition 4

Notice that

$$\lim_{\rho_i \to 1} \Psi^y(\rho_i) = -\frac{1}{\sigma}\frac{(1-\gamma)(1-\delta_1)}{\delta_1} + \frac{\sigma-1}{\sigma}\frac{1}{1-\beta(1-\delta_2)^{\frac{1}{\sigma}}} - \frac{1}{1-\beta(1-\mu)}$$

which is finite, since $\delta_1 > 0$, $\beta(1-\delta_2)^{\frac{1}{\sigma}} < 1$ and $\beta(1-\mu) < 1$

The derivative of Ψ^y with respect to ρ_i evaluated at $\rho_i = 1$ is

$$\left.\frac{\partial \Psi^y}{\partial \rho_i}\right|_{\rho_i=1} = -\frac{1-\gamma}{\sigma}\left(\frac{1-\delta_1}{\delta_1}\right)^2 + \frac{\sigma-1}{\sigma}\frac{\beta(1-\delta_2)^{\frac{1}{\sigma}}}{\left[1-\beta(1-\delta_2)^{\frac{1}{\sigma}}\right]^2} - \frac{\beta(1-\mu)}{[1-\beta(1-\mu)]^2}$$
$$+(1-\gamma)\frac{(1-\delta_1)}{(\delta_1)^2}\left[\frac{\sigma-1}{\sigma}\frac{1}{1-\beta(1-\delta_2)^{\frac{1}{\sigma}}} - \frac{1}{1-\beta(1-\mu)}\right]$$

By setting, $\left.\frac{\partial \Psi^y}{\partial \rho_i}\right|_{\rho_i=1} = 0$ we obtain an implicit expression for σ^*:

$$\sigma^* = 1 + \frac{\left[1-\beta(1-\delta_2)^{\frac{1}{\sigma^*}}\right][1-\beta(1-\mu)]}{\beta(1-\delta_2)^{\frac{1}{\sigma^*}} - \beta(1-\mu)} \frac{(1-\gamma)\frac{1-\delta_1}{\delta_1}\left[1-\delta_1+\frac{1}{1-\beta(1-\mu)}\right] + \delta_1\frac{\beta(1-\mu)}{[1-\beta(1-\mu)]^2}}{(1-\gamma)\frac{1-\delta_1}{\delta_1} + \delta_1\frac{1-\beta(1-\delta_2)^{\frac{1}{\sigma^*}}\beta(1-\mu)}{\left[1-\beta(1-\delta_2)^{\frac{1}{\sigma^*}}\right][1-\beta(1-\mu)]}}$$

Therefore, $\left.\frac{\partial \Psi^y}{\partial \rho_i}\right|_{\rho_i=1} < 0$ iff $\sigma < \sigma^*$ and $\left.\frac{\partial \Psi^y}{\partial \rho_i}\right|_{\rho_i=1} > 0$ iff $\sigma > \sigma^*$. This proves 4.b and the second part of 4.a. To prove 4.c, and the first part of 4.a, we set $\Psi^y(1) = 0$ and solve for σ to obtain and implicit expression for σ^{**} :

$$\sigma^{**} = 1 + \frac{\left[1-\beta(1-\delta_2)^{\frac{1}{\sigma^{**}}}\right][1-\beta(1-\mu)]}{\beta(1-\delta_2)^{\frac{1}{\sigma^{**}}} - \beta(1-\mu)}\left[(1-\gamma)\frac{1-\delta_1}{\delta_1} + \frac{1}{1-\beta(1-\mu)}\right]$$

Therefore, $\Psi^y(1) < 0$ iff $\sigma < \sigma^{**}$ and $\Psi^y(1) > 0$ iff $\sigma > \sigma^{**}$. It's easy to show that $\sigma^{**} - \sigma^* > 0$. Since Ψ^y is continuous in ρ_i, then $\exists \epsilon > 0$ such that the claims just proved for $\Psi^y(1)$ also apply to $\Psi^y(\rho_i)$, for $\rho_i \in (1-\epsilon, 1]$.

C Region of determinacy

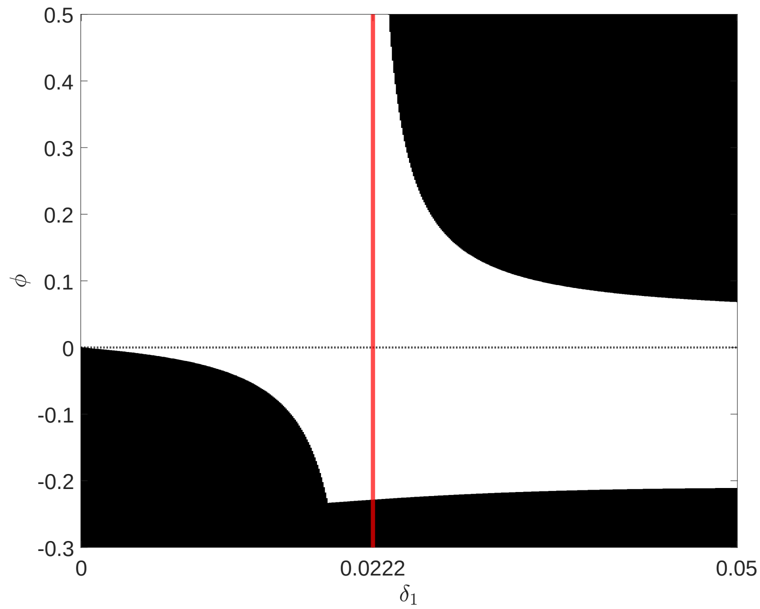

Figure C1: Visual representation of our model's region of determinacy (in white) as a function of ϕ and δ_1; red line represents our baseline choice for $\delta_1 = 1/45$. Other parameters calibrated as in footnote 23.

D Do the effects of monetary policy shocks vary with persistence?

An important feature of FLANK – distinguishing it from the standard New Keynesian model – is that rather transient monetary policy shocks do more to affect real activity in the conventional direction, than more persistent shocks (such as those associated with forward guidance). These contrasting predictions open the door to an empirical test, which is what we do here.

For the U.S., it has been observed (by, e.g., McKay and Wolf (2023)) that the monetary policy shock series by Romer and Romer (2004, "RR") rapidly leads to a short-lived peak in the Federal funds rate, while the shock of Gertler and Karadi (2015, "GK") captures a different dimension of monetary policy, more inclusive of "forward guidance", with the shock inducing a more delayed and persistent response in the policy rate.

To see whether these different shocks also yield different responses in activity, we generate IRFs by following Plagborg-Møller and Wolf (2021) in ordering the shock first in a recursive VAR (estimated at the monthly frequency) that also contains the Federal funds rate, the natural log of the CPI, the natural log of the commodity price index, and the natural log of Industrial

Production (our measure of real activity[39]). All data are taken from Ramey (2016), who – in turn – used the updated RR series of Wieland and Yang (2020).

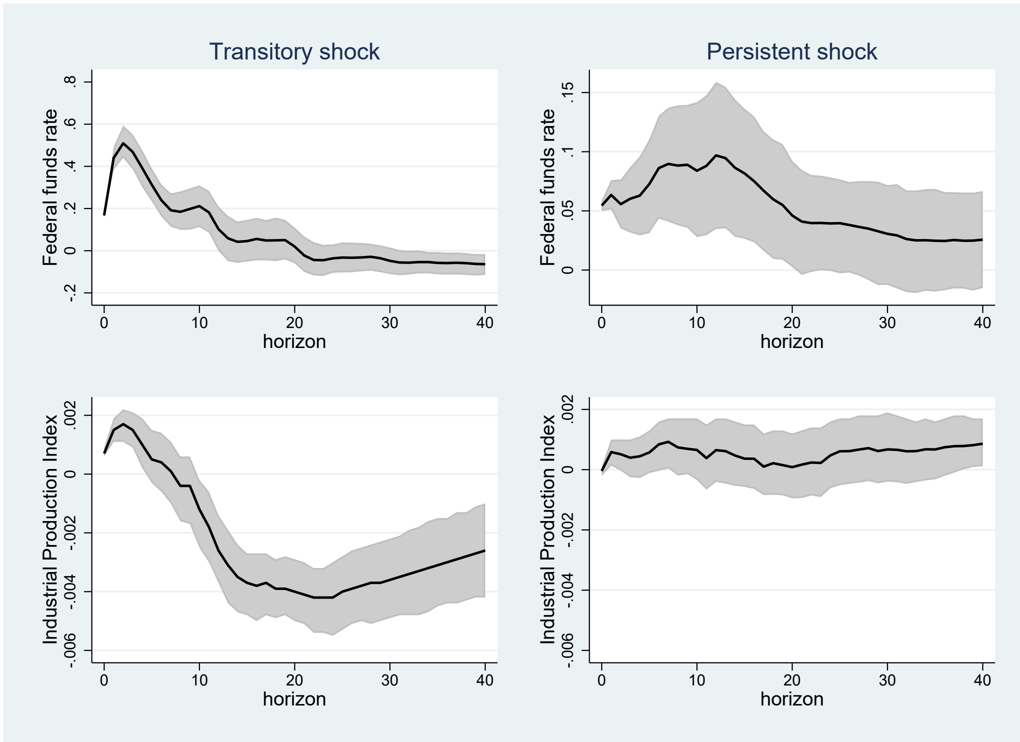

Figure D2: Response of Federal funds rate and industrial production index to monetary policy shocks of different persistence (transitory shock = RR; persistent shock = GK). VAR estimated at monthly frequency. Shaded areas represent 16th and 84th percentile confidence bands, obtained via bootstrapping.

As Figure D2 shows, the RR shock – which induces a more transient increase in the Federal funds rate compared to the GK shock – leads to a stronger contraction in real activity; using a different specification, McKay and Wolf (2023, Appendix C.2) obtain a similar finding, pointing towards some robustness of the bottomline conclusion.[40] While this is strongly at odds with the standard New Keynesian model (where the potency of monetary policy shocks is *increasing* in persistence – even under a discounted Euler equation), the apparent emergence of a "persistence-potency trade-off" is more in line with our FLANK model.

An alternative interpretation is to question the validity of, especially, the more persistent shock (which we simply took from Gertler and Karadi (2015)). It is however interesting to

[39]Looking at the response of the unemployment rate leads to the same conclusion.

[40]While McKay and Wolf (2023) find more evidence of the GK shock lowering activity, it is striking how – also in their specification – the RR shock is more potent on output, despite that impulse giving rise to a much smaller area under curve of the interest rate response. In the standard New Keynesian model, the strength of the activity response should be *increasing* in the area under the curve of the interest rate response (recall Proposition 3).

observe that other studies (using different shock series) have also found evidence to suggest that the potency of monetary policy shocks on activity decreases with persistence. Examples include Miescu (2023, for the U.S.), Swanson (2024, for the U.S.), and Braun et al. (2025, for the U.K.). A similar result is reported in Uribe (2022, for the U.S.), who takes a rather different approach to shock identification (not relying on high-frequency methods, but exploiting cointegrating relationships). FLANK is furthermore consistent with the observation that yield curve inversions tend to be followed by economic slowdowns (Harvey, 1988).[41] Our model suggests that such inversions might be more than "just" a recession signal, pointing to a potential causal link stemming from the notion that the combination of high short-term rates with lower long-term rates is contractionary on both ends of the curve.

Regardless of this, further empirical work aimed at identifying the causal impact of highly persistent monetary policy shocks would be desirable – also since it can help in the construction of policy counterfactuals (McKay and Wolf, 2023; Caravello et al., 2024).

E Extension with physical capital

In the extension with physical capital, good-producing firms operate the production function:

$$y_t = A \left(\ell_t\right)^{\eta} \left(k_{t-1}\right)^{1-\eta},$$

where $\eta \in (0, 1)$. All capital is owned by households who rent it to good-producing firms and invest to produce new capital. Investment is subject to a quadratic adjustment cost, such that producing inv_t new units of capital costs $inv_t + \frac{\iota}{2}\left(\frac{inv_t}{k_{t-1}} - \nu\right)^2 k_{t-1}$ units of output, with $\iota > 0$. Existing capital depreciates at rate $\nu \in [0, 1]$. Hence, its law of motion is $k_t = inv_t + (1-\nu)\,k_{t-1}$.

The optimal investment policy is:

$$inv_t = \left(\nu + \frac{q_t^k - 1}{\iota}\right) k_{t-1},$$

where q_t^k denotes the price of capital which is determined by the households' first order conditions:

$$1 = \mathbb{E}_t \left[\Lambda^r_{t,t+1} \frac{u_t + (1-\nu)\, q^k_{t+1}}{q^k_t}\right],$$

$$1 = \mathbb{E}_t \left[\Lambda^w_{t,t+1} \frac{u_t + (1-\nu)\, q^k_{t+1}}{q^k_t}\right],$$

[41] Also see Ang et al. (2006), who find that short-term rates have most predictive power when it comes to forecasting future GDP. This is again in line with our FLANK model, which implies that the short-term rate bears the least ambiguous relation to activity.

where $u_t = \frac{1-\eta}{\eta}\frac{\epsilon-1}{\epsilon}\chi\left(\frac{y_t}{\vartheta A}\right)^{1+\frac{1+\varphi}{\eta}}\left(\frac{1}{k_{t-1}}\right)^{1+(1-\eta)\frac{1+\varphi}{\eta}}$ is the rental rate of capital. The returns on the portfolios of assets held by retirees and workers are:

$$r_{t+1}^{r} = r_{t+1} + \left[\frac{1+(1-\mu)\,q_{t+1}}{q_t} - r_{t+1}\right]\alpha_t^r + \left[\frac{1+(1-\nu)\,q_{t+1}^k}{q_t^k} - r_{t+1}\right]\breve{\alpha}_t^r,$$
$$r_{t+1}^{w} = r_{t+1} + \left[\frac{1+(1-\mu)\,q_{t+1}}{q_t} - r_{t+1}\right]\alpha_t^w + \left[\frac{1+(1-\nu)\,q_{t+1}^k}{q_t^k} - r_{t+1}\right]\breve{\alpha}_t^w,$$

where α_t^j denotes the share of household-j wealth invested in long-term bonds and $\breve{\alpha}_t^j$ the share invested in capital. Market clearing in the asset markets requires:

$$q_t b^g = \vartheta\alpha_t^w a_t^w + (1-\vartheta)\,\alpha_t^r a_t^r,$$
$$q_t^k k_t = \vartheta\breve{\alpha}_t^w a_t^w + (1-\vartheta)\,\breve{\alpha}_t^r a_t^r,$$
$$0 = \vartheta\left(1-\alpha_t^w-\breve{\alpha}_t^w\right)a_t^w + (1-\vartheta)\left(1-\alpha_t^r-\breve{\alpha}_t^r\right)a_t^r,$$

while goods market clearing implies:

$$y_t = \frac{\vartheta c_t^w + (1-\vartheta)\,c_t^r + inv_t + \frac{\iota}{2}\left(\frac{inv_t}{k_{t-1}}-\nu\right)^2 k_{t-1}}{1-\frac{\theta}{2}\left(\pi_t-\bar{\pi}\right)^2}.$$

Finally, the real marginal cost of production is $\frac{\chi}{\eta}\left(\frac{y_t}{\vartheta A}\right)^{\frac{1+\varphi}{\eta}}\left(\frac{1}{k_{t-1}}\right)^{(1-\eta)\frac{1+\varphi}{\eta}}$. Hence, the Phillips curve becomes:

$$\left(\pi_t-\bar{\pi}\right)\pi_t = \lambda\left[\frac{\chi}{\eta}\left(\frac{y_t}{\vartheta A}\right)^{\frac{1+\varphi}{\eta}}\left(\frac{1}{k_{t-1}}\right)^{(1-\eta)\frac{1+\varphi}{\eta}} - 1\right] + \mathbb{E}_t\left[\Lambda_{t,t+1}^w\left(\pi_{t+1}-\bar{\pi}\right)\pi_{t+1}\frac{y_{t+1}}{y_t}\right].$$

All other equations remain unchanged.

For a zero inflation target ($\bar{\pi}=1$) and $\tau^r=0$, the steady-state real rate r solves:

$$\frac{y-\nu k}{r-\left[(1-\delta_2)\,\beta r\right]^{\frac{1}{\sigma}}}\frac{1+\frac{\delta_1[(1-\delta_2)\beta r]^{\frac{1}{\sigma}}}{1-(1-\delta_2)[(1-\delta_2)\beta r]^{\frac{1}{\sigma}}}}{\left[\frac{1-\beta(1-\delta_1)r}{\beta\delta_1 r}\right]^{\frac{1}{\sigma}} + \frac{\delta_1}{1-(1-\delta_2)[(1-\delta_2)\beta r]^{\frac{1}{\sigma}}}} = \frac{b^g}{r-1+\mu} + k,$$

where:

$$k = (\eta)^{\frac{1}{1+\varphi}}\left(\frac{\epsilon-1}{\epsilon}\frac{1-\eta}{r-1+\nu}\right)^{\frac{1}{\eta}},$$
$$y = \vartheta A\,(\eta)^{\frac{1}{1+\varphi}}\left(\frac{\epsilon-1}{\epsilon}\frac{1-\eta}{r-1+\nu}\right)^{\frac{1-\eta}{\eta}}.$$

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