BIS Working Papers

No 1201

Crypto Exchange Tokens

by Rodney Garratt, Maarten R.C. van Oordt

Monetary and Economic Department

July 2024

JEL classification: G10, G12, G18

Keywords: Asset pricing, Cryptocurrencies, Exchanges, Market manipulation

ISSN 1020-0959 (print)
ISSN 1682-7678 (online)

Crypto Exchange Tokens*

Rodney Garratt†
Bank for International Settlements

Maarten R.C. van Oordt‡
Vrije Universiteit Amsterdam
Tinbergen Institute

July 2024

Abstract

Crypto exchange tokens are an important funding source for centralized crypto exchanges, and they have been at the core of some of the biggest disruptions in the crypto industry. We develop a tractable model for the exchange rates of crypto exchange tokens that incorporates user demand, investment demand, and commonly observed pledges by exchanges to buy back tokens. We derive closed-formed solutions for the valuation of exchange tokens and the time required to fulfill the pledge. Buyback pledges increase the amount of funding raised by selling tokens. However, the additional amount raised is always less than the discounted cost of the buyback pledge. Future price manipulation by investors can further increase the cost of the buyback pledge.

Keywords: Asset pricing, Cryptocurrencies, Exchanges, Market manipulation.
JEL Codes: G10, G12, G18.

*We are grateful to Jonathan Chiu and participants in a seminar at the Bank of Canada for helpful comments and suggestions. Views expressed do not necessarily reflect those of the Bank for International Settlements.

†Email: rodney.garratt@bis.org
‡Email: m.van.oordt@vu.nl

I. Introduction

Exchange tokens are blockchain-based assets issued by crypto exchanges. These tokens may promise holders discounts on transaction fees, access to certain platform services and higher staking rewards. Exchange tokens have at times had a combined market value that exceeds 100 billion USD. While understudied in the literature, exchange tokens form the backbone of several of the largest crypto exchanges. They are a major source of funding that was crucial to the launch or continued operation of exchanges. Exchange tokens have also been at the core of some of the biggest disruptions in the crypto industry. For example, the LEO token was launched to quickly raise funds after losses revealed in court filings by the New York Attorney General shook confidence in the Bitfinex platform. Within two weeks after the release of its white paper, the platform claimed to have successfully raised $1 billion, more than the losses it incurred when trying to move funds through a Panamanian payment processor (New York AG, 2019; Bitfinex, 2019b). Moreover, the FTT token played a central role in the recent collapse of the once-celebrated and now-bankrupt FTX platform that owed its customers over 8 billion USD.

Exchange tokens typically involve buyback pledges whereby the issuer promises to buy back tokens using a share of their revenues.[1] Such pledges are designed to make exchange tokens more attractive to investors by reducing future token supply and driving price appreciation. Buyback pledges attempt to support the price of an exchange token by impacting future supply. Assuming buyback pledges are credible (i.e., the issuer is not simply defrauding investors), then why is this preferred to limiting the supply of the initial issuance? Under what conditions would an exchange commit to a buyback program? Who benefits and what are potential pitfalls when making such pledges?

[1]For example, at the time issuance, the Bitfinex exchange pledged to use more than a quarter of its gross revenue to buy back LEO tokens at the then-prevailing market rate until no tokens would be in commercial circulation (Bitfinex, 2019a). An overview of buyback pledges is provided in Section III.

This paper develops a tractable model for the exchange rates of crypto exchange tokens. The environment we study incorporates key aspects of the demand side and the supply side of this market. On the demand side we model both the user demand for exchange tokens and demand by (speculative) investors who have no use for the tokens but buy them in hopes of price appreciation. On the supply side, we include the pledge by the exchange to buy back a certain number of tokens. The environment also captures dynamic aspects including the potential future growth of the trading platform as well as the risk of the exchange defaulting on its promises to offer discounts and to buy back tokens (as was the case with the FTX-issued FTT tokens).

We begin the analysis by deriving closed-form solutions for the prices of exchange tokens as well as the time the exchange needs to buy back the promised number of tokens using the pledged resources. The equilibrium prices during the buyback phase are governed by either a *utility*-regime or an *investment*-regime. In the former regime, only users that derive utility from the benefits of exchange tokens hold them. In the latter regime, tokens are also held by investors that seek to profit from price appreciation.

The utility-regime arises in situations where the exchange pledges relatively few resources to buy back tokens. The buybacks occur at a relatively slow pace resulting in a limited decline in the number of circulating tokens over time. The resulting rate of appreciation in the token price will be too modest to entice investors to hold the tokens. Only users who obtain utility from the tokens will be willing to hold them. In equilibrium, the appreciation in token prices will be governed by both the growth in user demand and the speed of buybacks provided that the exchange does not collapse. Once the exchange completes the buyback program, the expected rate of token appreciation will be governed solely by growth in user demand.

The investment-regime arises in situations where the exchange pledges a relatively large amount of resources to buy back tokens. The faster pace of buybacks results in a relatively swift decline in the circulating number of tokens. The faster decline in the circulating supply of tokens attracts investors who anticipate a higher rate of appreciation. Both users and investors will be willing to hold the tokens as a result. Investors will be willing to invest in the tokens until the exchange completes the promised buyback program of the tokens. Once the exchange completes the buyback program, the tokens are no longer appealing to investors and their rate of appreciation is determined by growth in user demand alone. Until that moment is reached, the rate of token appreciation is determined by the return that investors require on capital as well as the default probability of the exchange.

Whether it is the utility-regime or the investment-regime that determines the price path of the token during the buyback phase depends on how the amount of resources that the exchange pledges to buy back tokens compares to the user demand for tokens. For investors to be willing to hold the tokens, it must be the case that the amount of resources pledged for buybacks as a percentage of user demand exceeds the sum of the exchange's cost of capital and its default intensity minus the user growth rate.

The length of the buyback phase in both regimes decreases in the amount of resources that the exchange pledges for buybacks and increases in the share of tokens that the exchange promises to buy back. The buyback phase never ends in the special case where the exchange promises to repurchase all the tokens. In this situation, the exchange is forced to carry on spending the pledged resources on a continually dwindling number of tokens at ever-increasing prices.

Our analysis allows us to assess the costs to the platform of raising funds through the issuance of exchange tokens with buyback pledges. Buyback pledges may increase the amount of funds the exchange can raise by making the tokens more attractive to investors, but we identify two potential pitfalls. First, we show that the additional funds that can be raised due to a buyback pledge are less than the discounted cost of buybacks. In other words, the cost of capital of the additional funds raised due to a buy back pledge is higher than the return required by investors. This wedge may cause conventional capital markets to be a more cost-effective source of funding than including buyback pledges for exchanges that have access to traditional capital markets. Second, we show that there is the potential for investors to take advantage of the commitment of the issuer to buy back tokens by manipulating the supply of tokens the issuer can repurchase.[2] Manipulation of the token supply by investors can increase the cost of buyback pledges to exchanges even further, and underlines the benefits of conventional capital markets to raise funds for exchanges with access to funding on these markets.

A large investor could potentially benefit by restricting the number of tokens available to other market participants. We show that it can be profitable for an investor to permanently freeze (or "burn") part of her token holdings so that the exchange will be prevented from ever completing the buyback program. Such an action will force the exchange to continue the buyback program ad infinitum, while continually driving up the price and the market capitalization of the tokens. Our results suggest that exchange tokens with little user demand and large buyback promises may be vulnerable to being exploited by an investor implementing this strategy. We show that exchanges can prevent its token from being vulnerable to such an attack by following a rule-of-thumb where it limits the share of tokens it pledges to buy back to one-half.

The remainder of the paper is organized as follows. Section II discusses related literature. Section III provides a brief overview of the realm of crypto exchange tokens. Section IV presents the model. Section V describes the equilibrium exchange rate path for exchange tokens with and without investors. Section VI considers the cost of the buyback program, first without price manipulation by investors, and then under different scenarios where price manipulation may occur, and explains why buybacks are an inefficient form of funding. Section VII considers an extension where users have elastic demand for token services and factor investment motives into their demand for tokens. Section VIII provides concluding remarks. Proofs are provided in the Appendix.

[2] This type of manipulation has aspects in common with a short squeeze in finance.

II. Related literature

The academic literature has made considerable progress in terms of understanding cryptocurrency exchange rates. A variety of theoretical models have been used to explain the dynamics of cryptocurrency exchange rates (Athey et al., 2016; Biais et al., 2023; Bolt and Van Oordt, 2020; Chiu and Koeppl, 2022; Garratt and Van Oordt, 2023; Garratt and Wallace, 2018; Karau and Moench, 2023; Pagnotta, 2022; Prat et al., 2024; Schilling and Uhlig, 2019). A substantial body of theoretical research has examined the potential of funding firms or platforms with cryptocurrencies or tokens (Bakos and Hałaburda, 2022; Chiu and Wong, 2022; Cong et al., 2021, 2022; Garratt and Van Oordt, 2022; Gryglewicz et al., 2021; Lee and Parlour, 2022; Malinova and Park, 2023a; Rogoff and You, 2023; Sockin and Xiong, 2023). To the best of our knowledge, we are the first to explore the valuation of a crypto token where the issuer commits resources to buy back a certain number of tokens as well as the opportunity for strategic investors to take advantage of that commitment.

Several recent papers on tokenomics highlight features that are closely related to aspects of the model in the present paper. Cong et al. (2022) study optimal token buybacks ("burns") in an environment without investors but where the ensuing appreciation in token prices may stimulate user demand for the tokens. Our baseline model separates user demand from investor demand although we also consider environments where users have elastic demand for token services and factor investment motives into their demand for tokens. Another closely related paper is the study by Prat et al. (2024) which analyzes token prices in an environment that allows for both user-demand and investor-demand for tokens. They also recognize that equilibrium token prices can be characterized by either a utility-regime or an investment-regime depending on the expected rate of appreciation (an earlier study making a similar observation in a single-period model is Bolt and Van Oordt, 2020). A distinction is that Prat et al. (2024) do not study an environment where the issuer pledges resources to buy back tokens while we do. Buybacks are an important characteristic of exchange tokens that is key to understanding the potential for large price fluctuations driven by strategic investors. As far as we are aware, we are the first to show that the equilibrium with nonstrategic investors may not be robust to manipulation by a strategic investor who incorporates the impact of her own actions on token prices.

Finally, the scope of this paper is limited to tokens issued by centralized crypto exchanges. Decentralized crypto exchanges that operate through smart contracts on distributed ledgers offer an interesting alternative for trading crypto tokens (Aoyagi and Ito, 2021; Lehar and Parlour, forthcoming; Malinova and Park, 2023b). Decentralized crypto exchanges often issue governance tokens that provide voting rights regarding changes in the smart contracts that define the operating procedures of those exchanges. These tokens have a different type of function than the tokens issued by centralized exchanges that are studied here.

III. Background

A substantial share of the world's largest centralized crypto exchanges have launched their own tokens to raise funding or to foster customer loyalty. Table 1 summarizes hand-collected key characteristics of crypto exchange tokens sorted by market capitalization. The list includes tokens issued by six out of the ten largest centralized crypto exchanges by trading volume.[3] The market capitalization of the tokens is dominated by Binance's BNB token which is also the token with the longest trading history (starting in September 2017).

All exchanges in the table offer discounts on trading fees for owners of their tokens, although the requirements for traders to qualify for discounts varies across exchanges. The most common scheme is to offer traders a discount if they use the tokens to pay for fees on the trading platform. Another popular scheme is to require traders to maintain a certain *dollar*-amount of tokens to qualify for discounts. Finally, some exchanges provide discounts based on the number of tokens that a trader holds. This latter approach is sometimes offered

[3]Based on the ten largest exchanges by trading volume as reported by coinmarketcap.com (as of September 1, 2023). See Aloosh and Li (forthcoming) and Cong et al. (2023) for a cautionary tale about statistics on trading volumes on centralized crypto exchanges.

Table 1: Key Characteristics of Crypto Exchange Tokens

Exchange	Token	Market cap	Buyback target	Pledged resources for buybacks	To qualify for discounts	Source
Binance	BNB	39,320	50%	Profit (20%)	Use to pay fees	White paper
FTX	FTT	3,410	50%	Transaction fees (33%)	Hold dollar-amount	Both
Bitfinex	LEO	3,340	100%	Gross revenue (27%)	Hold dollar-amount	White paper
OKX	OKB	1,520	Unknown	Spot transaction fees (30%)	Hold tokens	Web page
Huobi	HT	976	Unknown	Revenue (20%)	Use to pay fees, hold (dollar-)amount	Web page
Kucoin	KCS	646	50%	Net profit (10%)	Hold tokens	Web page
Gate	GT	327	Unknown	Net profit (15%)	Use to pay fees, hold tokens	Web page
Bidget	BGB	254	Unknown	Not specified	Use to pay fees	White paper
MEXC	MX	82	90%	Profit (40%)	Use to pay fees	Web page
Wazirx	WRX	52	10%	Not specified	Use to pay fees	White paper
Tokocrypto	TKO	25	10%	Revenue (10%)	Use to pay fees, hold tokens	Both
Bitmart	BMX	17	50%	Trading fees income (20%)	Use to pay fees	White paper
Bibox	BIX	1	60%	Net profit (25%)	Use to pay fees	White paper

Note: Market capitalization in million USD as of 31 December 2022 (FTT: as of 31 October 2022). The buyback target is the fraction of coins that the exchange pledges to repurchase and burn. Sources: coinmarketcap.com and hand-collected data.

to provide additional discounts on top of those where traders pay for fees with the exchange token. Some exchanges also aim to increase the attractiveness of their tokens by offering alternative benefits that may be appealing to particular user segments such as lotteries or advance access to offerings of new tokens.

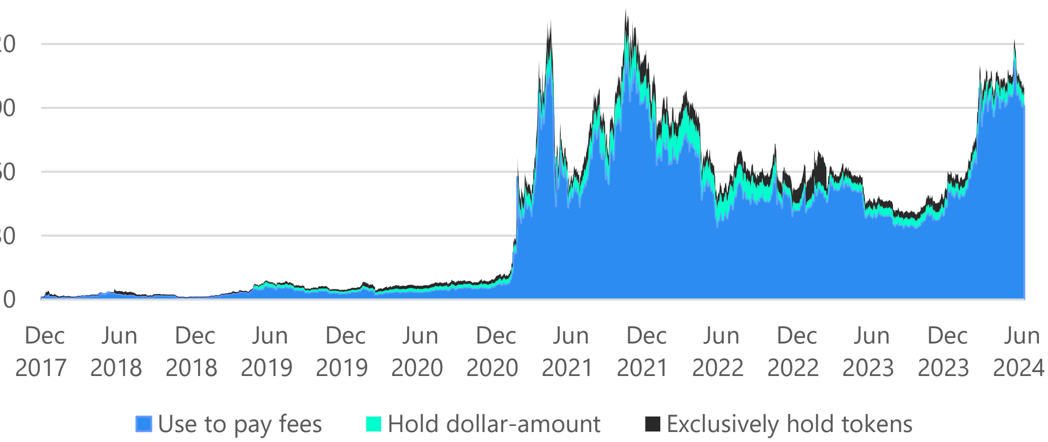
Almost all exchanges have pledged resources to buy back and burn tokens in a somewhat concrete manner. The pledged resources are, in all such cases, in proportion to the activity on the trading platform or its financial success. The platforms use various measures to calculate the amount of resources pledged for buy backs, ranging from a share of the transaction fees (e.g., FTX and OKX) to a share of the profits (e.g., Binance and Kucoin). Moreover, the majority of exchanges report explicit targets in terms of the total share of tokens that they intend to buy back and burn. This percentage ranges from ten percent for some of the smaller tokens to a hundred percent in case of the aforementioned LEO tokens.[4]

[4] Binance wrote in their white paper "Every quarter, we will use 20% of our profits to buy back BNB and destroy them, until we buy 50% of all the BNB (100MM) back. All buy-back transactions will be announced on the blockchain. We eventually will destroy 100MM BNB, leaving 100MM BNB remaining." (Binance, 2018, p. 9); FTX wrote in their white paper "One third of all fees generated on FTX will be used for an FTT repurchase, until at least half of all FTT is burned. Any FTT bought this way will be burned" (FTX, 2020, p. 7); Bitfinex wrote in their white paper "On a monthly basis, iFinex and its affiliates will buy back LEO from the market equal to a minimum of 27% of the consolidated gross revenues of iFinex (exclusive of Ethfinex) from the previous month, until no tokens are in commercial circulation. Repurchases will be made at then-prevailing market rates." (Bitfinex, 2019a, p. 14).

Figure 1: Evolution of Market Capitalization of Crypto Exchange Tokens

(a) As a percentage of Bitcoin's market capitalization

(b) In billion USD.



Note: The aggregate market capitalization of the crypto exchange tokens listed in Table 1. The colors in panel (b) indicate how owners of tokens can qualify for discounts on trading fees. Source: Coingecko.com.

Figure 1 shows the evolution in the market capitalization of crypto exchange tokens. The market capitalization of crypto exchange tokens was remarkably stable at a level of around 5 percent when expressed as a percentage of bitcoin's market capitalization during 2019-2020 (Figure 1a). Early 2021, the market capitalization exploded from a level of around 5 billion USD to a level exceeding a 100 billion USD (Figure 1b). This increase was almost exclusively due to the strong appreciation of existing crypto exchange tokens during a period where the bitcoin exchange rate skyrocketed. The market cap of crypto exchange tokens in terms of bitcoin's market capitalization approximately doubled to around 10 percent during this period. After mimicking the double peak in the bitcoin price in 2021, the market value of crypto exchange tokens dropped along with the exchange rate of bitcoin. The value of crypto exchange tokens has increased since the second half of 2023 in tandem with the increase in the exchange rate of bitcoin. The value of crypto exchange tokens reaches a final level in the chart of around 100 billion USD.

IV. Model

Many crypto exchange tokens share two main characteristics. First, the holders of exchange tokens are promised certain benefits by the issuers. Second, the cryptocurrency exchange often pledges resources to buy back some of the tokens at the prevailing market rate in order to withdraw these tokens from circulations. We consider an environment that incorporates these main characteristics.

Time is continuous. The exchange issues tokens at time $t = 0$. The initial number of tokens that the platform issues is $M > 0$. The exchange rate or market price of the tokens at time t is denoted by $S(t)$. No new tokens are added and the cryptocurrency exchange promises to repurchase $U \geq 0$ tokens over time at the prevailing market prices and burn them (i.e., permanently remove them from circulation). The amount the exchange spends on buying back tokens is denoted by the dollar flow $Y^{\$}(t)$. The buybacks are assumed to occur evenly over time at infinitely small time intervals (in practice, exchanges buy back tokens at a lower frequency, such as a weekly or quarterly frequency). The endogenous number of tokens that the exchange repurchases and burns between time t_1 and t_2, if the exchange does not collapse, is denoted by $W(t_1, t_2)$. The value $W(0, t)$ identifies the total number of tokens that the exchange has repurchased and burned by time t. The endogenous moment in time at which the exchange completes its buyback program is denoted by T.

The benefits offered by the issuer to token holders generate a utility-demand for the tokens that may vary over time. We denote this utility-demand as $X^{\$}(t)$, where the superscript indicates that the utility value is measured in dollars. We treat $X^{\$}(t)$ as exogenous.[5] Some exchanges require a user's holdings to exceed a certain threshold amount in dollar terms in order to qualify for benefits such as discounts (e.g., Bitfinex and the now-defunct FTX). Users of these exchanges have an immediate incentive to ensure that their token holdings do not fall below the threshold dollar amount. Many other exchanges offer discounts if users pay for fees by paying with the tokens they issued (e.g., Binance and Huobi). For those tokens, the utility-demand in terms of dollars will depend on the aggregate dollar amount of the transaction fees that are paid for with the tokens divided by the velocity of tokens that traders hold in their accounts with the exchange to pay for those fees.

Tokens that are neither bought back by the exchange, nor held by users for the utility they provide, are held by investors. Investors are risk-neutral profit maximizers who evaluate the present value of expected cash flows from their investments with a continuous time cost of capital $r > 0$. Investors are assumed to be nonstrategic in the sense that individual investors take token prices as given and do not incorporate the impact of their own actions on current and future token prices. We denote the aggregate number of tokens held by investors at time t by $Z(t) \geq 0$. The non-negativity constraint comes from the fact that, in the aggregate, investors cannot hold a short position in the tokens since this would imply an increase in the supply of tokens.

Activity on the trading platform is assumed to evolve at a fixed growth rate $g < r$ until the platform collapses. Activity on the platform at time t is $A(t) = e^{gt}$ so long as no collapse has occurred before time t, and $A(t)$ is arbitrarily close to 0 otherwise.[6] Without loss of generality, we normalize activity on the platform such that $A(0) = 1$. Activity affects both the utility-demand for tokens and the resources available for token buy backs. We assume that the utility-demand and the resources spent on buybacks are proportional to the activity on the exchange. In particular, we assume that the utility-demand for the tokens

[5]The underlying dollar demand of users for tokens can originate in various ways, as shown in Table 1.

[6]This assumption together with the no-Ponzi scheme condition in Definition 1 ensures that there are no rational bubble equilibria; see Van Oordt (2024).

issued by the platform is given by $X^\$(t) = X^\$ A(t)$ and the amount spent on buybacks is given by $Y^\$(t) = Y^\$ A(t)$. The exogenous constants $X^\$ > 0$ and $Y^\$ \geq 0$ are the initial dollar-values of the utility-demand and the pledged buyback resources. Finally, we assume that the probability that the trading platform is still in operation at time t is given by $e^{-\lambda t}$, where $\lambda \geq 0$ denotes the default intensity of the platform.

A. *Equilibrium definition*

User demand for tokens is pinned down, at any time t, by the exogenously specified process that determines activity on the platform and the exchange rate. Market clearing at each time t, which requires that the token supply equals the token demand by users and investors, can be written as

$$\underbrace{M - W(0,t)}_{\text{Token supply}} = \underbrace{\frac{X^\$(t)}{S(t)} + Z(t)}_{\text{Token demand}}, \tag{1}$$

where $W(0,t) = \int_0^t (Y^\$(t)/S(t))dt$. Determination of a market equilibrium requires finding a well-behaved exchange rate path such that, at each time t, investors optimally hold any existing tokens that are not held by users.

We now state the following:

Definition 1 *An equilibrium of the exchange token market consists of paths for $S(t)$ and $Z(t)$ such that*

(i) the market clearing condition in (1) holds for any t;

(ii) investors have no incentive to deviate from the path in any t; and

(iii) the no-Ponzi scheme condition $\mathbb{E}_t \int_t^\infty -\frac{\partial Z(\tau)}{\partial \tau} e^{-r\tau} S(\tau) d\tau \geq 0$ holds for any $t > 0$.

The no-Ponzi scheme condition requires that the net present value of the expected aggregate cash flows of investing in the token must be non-negative in equilibrium.[7]

V. Equilibrium Analysis

A. *Graphical representation of market clearing condition*

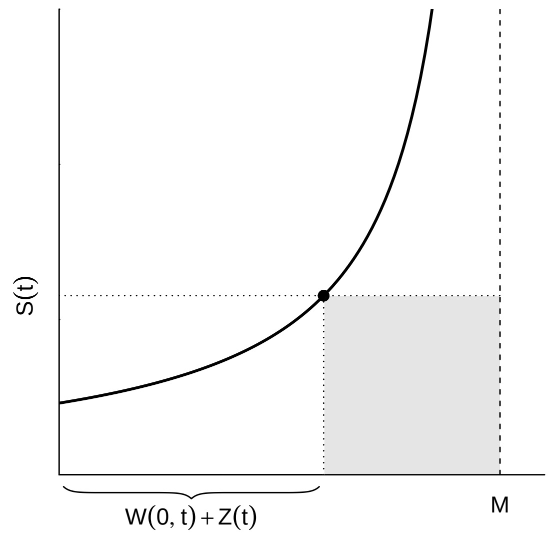
Before characterizing the equilibrium outcomes, we provide a graphical representation of the mechanics implied by the market clearing condition. Figure 2 illustrates the market clearing condition in (1) for a given value of $X^{\$}(t)$, which is represented by the shaded rectangle in the figure. The upward-sloping curve can be interpreted as the supply curve of tokens by users to investors and the trading platform. The total number of tokens held by investors and that where bought by the trading platform at time t equals $W(0,t)+Z(t)$. The users hold the remainder of the tokens, i.e., $M-W(0,t)-Z(t)$. Users maintain a total dollar balance of $X^{\$}(t)$. If the exchange rate appreciates, then users are willing to supply investors and the exchange with additional tokens because users need fewer tokens to maintain the same dollar balance of $X^{\$}(t)$. This brings about the upward curvature in the relationship between the exchange rate and the number of tokens that are held for investment purposes or by the exchange.

B. *Equilibrium exchange rate with and without investors*

The equilibrium level of the exchange rate depends on whether the buy back pledge of the exchange is large enough to entice investors to hold the tokens. The level of pledged resources necessary to convince investors to hold tokens increases in the required return on

[7]Condition (iii) is less restrictive than the transversality condition that is typically imposed. Differently from the typical transversality condition, it allows the exchange rate to appreciate forever at rate r when the exchange buys back tokens ad infinitum. The condition rules out rational bubble equilibria where ever-appreciating exchange rates are driven by speculators who continue to purchase tokens without expectation to ever profit in the aggregate from selling them.

Figure 2: The Exchange Rate and Market Clearing



Note: The solid curve shows how the token exchange rate varies with number of tokens available to users for a given level of $X^{\$}(t)$. The area of the shaded rectangle equals $X^{\$}(t)$ for any point on the curve.

capital and the failure rate of the platform, and decreases in the growth rate of the platform, as shown in the following proposition.

Proposition 1 *Consider an environment with nonstrategic investors. In equilibrium, investors hold tokens initially if and only if $Y^{\$} > (r + \lambda - g)X^{\$}$. If investors hold tokens initially, then, assuming no collapse occurs before time $t \in [0, T]$,*

$$S(t) = e^{-(r+\lambda)(T-t)} \frac{X^{\$} e^{gT}}{M - U},$$

where

$$T = \frac{1}{r + \lambda - g} \log \left(\frac{U}{M - U} \frac{(r + \lambda - g)X^{\$}}{Y^{\$}} + 1 \right).$$

If investors do not hold tokens initially, then, assuming no collapse occurs before time $t \in [0, T]$,

$$S(t) = e^{t(g + Y^{\$}/X^{\$})} \frac{X^{\$}}{M}$$

and

$$T = \frac{X^{\$}}{Y^{\$}} \log\left(\frac{M}{M-U}\right).$$

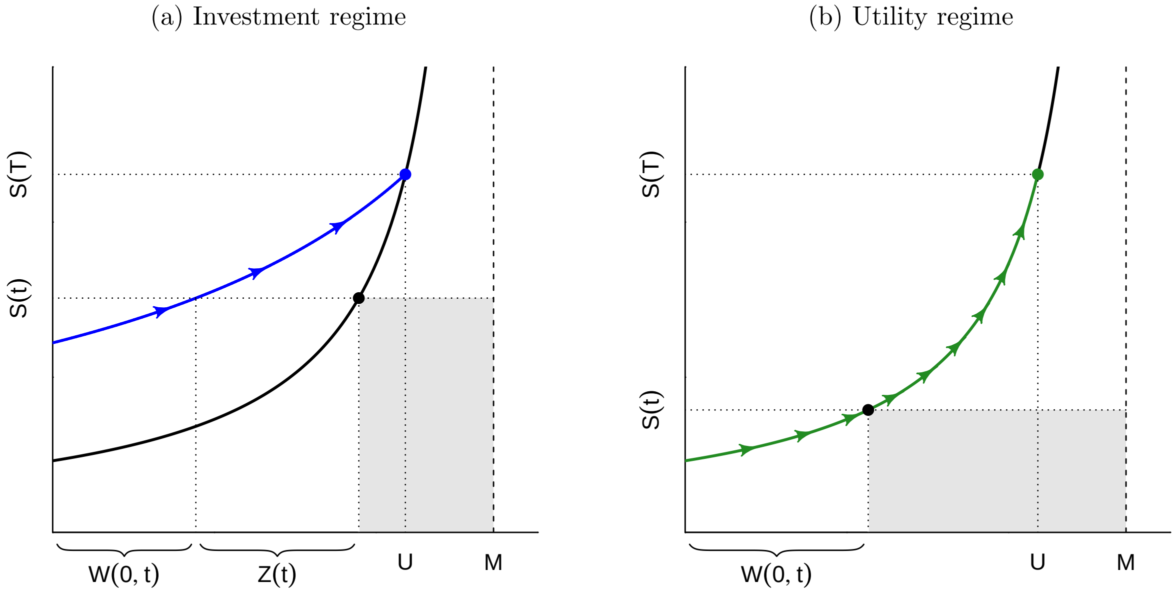
If a collapse occurs at time $\hat{t}$, then, regardless of whether investors were holding tokens, $S(t) = 0$ for $t \in [\hat{t}, T]$.

The proposition confirms the intuitive idea that the duration of the buyback program, T, increases when the number of tokens the exchange promises to buyback, U, is higher and when the exchange pledges a smaller amount of resources, $Y^{\$}$, to complete the buyback. Completion of the buyback program also takes longer with higher user demand, $X^{\$}$, due to the higher price at which the exchange has to buy back the tokens.

The evolution of the exchange rate without a crash in the two equilibrium scenarios in the proposition are illustrated in the two panels of Figure 3. The black curve in both panels shows how the equilibrium exchange rate $S(t)$ varies in response to the number of tokens available to users (as in Figure 2). The blue curve in panel (a) shows the equilibrium path of the exchange rate under the buyback program when investors choose to hold tokens. The green curve in panel (b) shows the equilibrium path of the exchange rate when investors do not hold tokens, and hence tokens are bought back from users only. The figures are drawn for the case of $g = 0$, which means zero growth of the platform. The black curve does not shift in this case, and hence changes in the exchange rate are driven entirely by buybacks, as indicated by the arrows. The blue curve in panel (a) lies above the black curve because the added demand from investors drives up the price of tokens starting at $t = 0$. The exchange finishes their pledge to buy back tokens at $t = T$, after which the exchange rate dynamics are driven solely by user demand.

One can distinguish between partial and full buyback programs. If the platform promises to buy back all of the tokens it issues (i.e., if $U = M$) we say the buyback program is full, otherwise, if $U < M$, we call it partial. The equilibrium for a full buyback program can be

Figure 3: Price Equilibrium without Crash



Note: The figure shows equilibrium price paths without a crash for $g = 0$. The black curve shows the equilibrium exchange rate without a buyback commitment. The blue and green curves show the equilibrium price paths with a commitment in the case where investors hold and do not hold tokens, respectively. The arrows reflects the number of tokens bought back by the issuer $W(0, t)$ over time.

obtained by taking the limit $U \to M$ in Proposition 1. It is summarized in the following corollary.

Corollary 1 *Suppose the buyback program is full. The exchange never finishes buyback program* ($T \to \infty$)*. If* $Y^\$ > (r + \lambda - g)X^\$$*, then the initial equilibrium exchange rate equals*

$$S(0) = \frac{1}{M} \frac{Y^\$}{r + \lambda - g}.$$

Otherwise, the initial equilibrium exchange rate equals $S(0) = X^\$/M$*.*

If investors choose to hold the tokens involved in a full buyback program, then the exchange rate of the tokens is reminiscent of the valuation of a perpetually growing cash flow under the Gordon growth model (Gordon, 1959). For this special case, it doesn't matter for the valuation of the tokens whether pledge resources are used to repurchase coins, or whether pledged resources would be distributed directly as a dividend to owners of the tokens. If investors choose not to hold the tokens, then the initial exchange rate simply reflects the ratio between the utility-demand and the number of tokens.

VI. Cost of the Buyback Program to the Platform

A. Wedge between cost of capital and required return on capital

The model can be used to assess how the additional funds an exchange can raise by making a buyback pledge compare to the expected cost of the buyback program to the exchange. The amount of funds that the exchange raises by selling tokens with a buyback pledge equals the total number of tokens multiplied by equilibrium exchange rate in Proposition 1, i.e., $MS(0)$. From the equilibrium exchange rates in the proposition, one can also calculate that the revenue from selling tokens is exactly $X^{\$}$ under the counterfactual where the exchange would not buy back any tokens ($U = 0$). Hence, the incremental revenue accrued from the buyback pledge equals $MS(0) - X^{\$}$. Let C denote the expected discounted cost of the pledged resources over the entire horizon of the program. This amount can be calculated as $C = \int_0^T e^{-rt}\mathbb{E}Y^{\$}(t)dt$. Comparing the cost of the buyback pledge to the incremental amount of funds raised yields the following result.

Proposition 2 *The additional funds an exchange can raise by offering a buyback pledge are less than its expected discounted cost.*

A direct implication of Proposition 2 is that the cost of capital for the additional funds that the exchange can raise by including a buyback pledge in its tokens sales exceeds the required return by investors. Hence, buyback pledges should be avoided if the exchange has access to efficient capital markets where funds could be raised at a cost of capital equal to the required return by investors. Table 1 shows that most sales of crypto exchange tokens include buyback pledges. One potential reason may be market access, or lack thereof. The major exchanges that issued exchange tokens are or were operated from offshore jurisdictions

(CoinGecko, 2023), or even refuse to disclose the location of their headquarters (Financial Times, 2023). This could make it difficult for them to access conventional capital markets. An exchange that lacks viable alternatives may find it appealing to include a buyback pledge if the exchange wishes to raise sufficient funds to build or expand its operations. Another potential reason why crypto exchanges might have opted for buyback pledges is that the required return by investors in crypto markets was lower than in conventional capital markets at the time of the token sales (or, alternatively, that crypto assets were "overpriced").

B. Additional costs under price manipulation

The cost of raising funds through a token sale with a buyback pledge may be exacerbated because of price manipulation. This subsection examines the potential for price manipulation by a large investor. Specifically, we look at whether an investor could benefit from taking advantage of the platforms commitment to buy back tokens by permanently freezing (or "burning") enough tokens so that the exchange never finishes the buyback program. The exchange will be forced to continue the buyback program ad infinitum, while continually driving up the price and the market capitalization of the tokens. First, we consider the case where a single large investor holds all of the coins not held by users. Second, we consider the case where multiple investors hold tokens and we establish what share of tokens a single investor would have to hold to be willing to engage in price manipulation. Finally, we consider the possibility of coalition building for the purposes of price manipulation.

1. Single large investor

The following result that applies to the situation where a single large investor owns all of the tokens not held by users.

Proposition 3 *Consider the situation where $Y^{\$} > (r + \lambda - g)X^{\$}$ and token prices and investment are according to the equilibrium in Proposition 1. Suppose all tokens not held by users are in possession of a single investor. It is feasible for the investor to burn $M - U$*

tokens if and only if

$$U > M \frac{Y^{\$}}{2Y^{\$} - (r + \lambda - g)X^{\$}}.$$

It is profitable to the investor to burn $M - U$ tokens if and only if

$$U > M \frac{Y^{\$}}{2Y^{\$} - 2(r + \lambda - g)X^{\$}}.$$

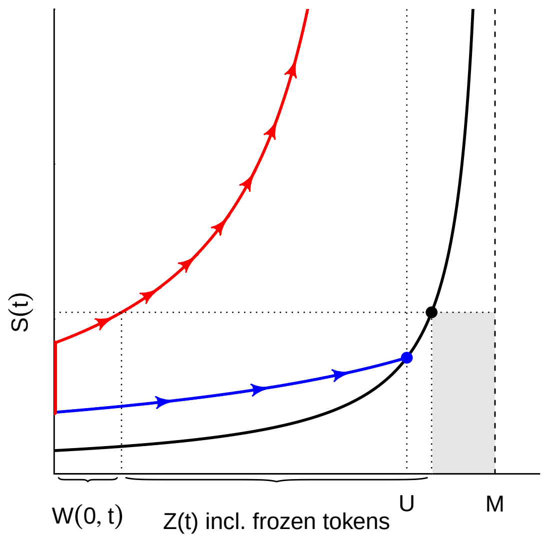
The exchange rate path if an investor that owns all tokens not held by users burns $M - U$ coins is reflected by the red curve in Figure 4. After the investor burns the tokens, the exchange rate jumps to a higher level. The new exchange rate path is equivalent to that under the equilibrium where the exchange intends to buy back all existing tokens in Proposition 4, except that the total number of existing tokens equals U rather than M. The level of the exchange rate on this path is above the equilibrium path indicated by the blue curve. The speed of buybacks is slower due to the higher exchange rate as indicated the length of the arrows. On the blue curve, the exchange finishes the buyback of U tokens in finite time. On the new path where the investor burns the tokens, the exchange never successfully finishes the program to buy back U tokens. The exchange rate continues to appreciate as the exchange buys back more and more of the remaining tokens, while fewer and fewer tokens are available to users.

2. One large investor, multiple smaller investors

The scenario where a single investor owns all tokens not held by users is probably unrealistic. Suppose instead that there are multiple investors. The following proposition determines the minimum share of tokens held by the largest investor that is required in order to make it still worthwhile for them to burn tokens.

Proposition 4 *Consider the situation where $Y^{\$} > (r + \lambda - g)X^{\$}$ and token prices and investment are according to the equilibrium in Proposition 1. Suppose the largest investor holds fraction ω_1 of all tokens held by investors at $t = 0$. Burning $M - U$ coins is profitable*

Figure 4: Price Path if Single Investor Destroys Coins



Note: The red curve shows the price path under the trading strategy described in Proposition 3 where a large investor permanently freezes $M-U$ tokens. The arrows reflect the number of tokens bought back by the issuer $W(0,t)$ over time. The blue curve reflects the exchange rate without manipulation as in Figure 3a with the vertical axis re-scaled. The figure assumes that $g=0$ and that no collapse of the issuer occurs.

for the largest investor if $\omega_1 > \omega^*$, *where*

$$\omega^* = \frac{MY^\$ - U(Y^\$ - (r+\lambda-g)X^\$)}{U\left(Y^\$ - (r+\lambda-g)X^\$\right)}.$$

Whenever a large investor burns tokens, other investors benefit as well. It is therefore interesting to ask whether the largest investor can convince other investors to join her when she is burning tokens. Having other players joining a token-burn is clearly better for the largest investor, because it allows the largest investor to burn less. Her payoff is $S'(0)\left[\omega_1 Z(0) - (M-U)\right]$ if she is the only player burning, which is less than the payoff of $S'(0)\left[\omega_1 Z(0) - (M-U)\omega_1/(\omega_1+\omega_2)\right]$ if the second-largest player burns as well. However, the second-largest player has no incentives to join the largest player, because her payoff will be smaller less if the largest investor would burn otherwise. The payoff of the second-largest player is $S'(0)\left[\omega_2 Z(0)\right]$ if the largest player burns alone, which is more than the payoff $S'(0)\left[\omega_1 Z(0) - (M-U)\omega_2/(\omega_1+\omega_2)\right]$ if she burns as well. Hypothetically, the largest player could threaten not to burn tokens if the second-largest player will not join her. However, this would not be a credible threat since it remains profitable for her to burn tokens if $\omega_1 > \omega^*$. Hence, if the holdings of the biggest investor are large enough, then it may prove challenging for her to convince the second-largest investor to join burning the tokens.

3. Many small investors

Suppose $\omega_1 < \omega^*$. Then no single investor holds sufficiently many tokens to profitably burn. There may still be a coalition that could burn tokens profitably. Let ω_2 denote the share of investor-held tokens owned by the second largest investor. Assuming that investors burn a proportional share of their investment holdings, we know from Proposition 4 that both the largest and the second-largest investor must be better off compared to no one burning if both burn and $\omega_1 + \omega_2 > \omega^*$.

One might pose the question whether it is practically possible for both players to coordinate the burning of the tokens on a blockchain without risking a scenario where the other player backs out. This may be less of a concern in the context of cryptocurrencies. The burn strategy of a coalition could be implemented with a standard multi-input transaction. Investor 1 can provide investor 2 with a multi-input transaction to a burn address that investor 1 has already signed so that only investor 2 needs to add her signature. The multiple inputs would contain the token balances that the investors in the coalition would burn in the agreed-upon strategy. The transaction cannot go through in its entirety unless both players sign it. If investor 2 declines to sign, then investor 1 could withdraw by broadcasting a single-input transaction that sends the balance from her own input address to another address of hers. Once this transaction is processed, the multi-input transaction would become invalid.

The logic above can be extended to a third investor for the case where the combined holdings of the first and the second largest investor are not big enough, and subsequently to a fourth investor, etc. After receiving the signed multi-input transaction from investor

1, investor 2 could add her signature and forward it to investor 3. The arrangement could work for any number of investors, at least in theory.

In practice, managing the communication among a large number of players may be difficult. Moreover, such communication may be interpreted as a form of collusion, which would cause the arrangement to fare into uncharted legal waters. The exchange might feel justified in changing the terms of the buyback program by reducing the number of tokens it burns by the number burned by investors. Detecting the manipulative scheme will be much harder in the scenario of a large single investor who may simply seem to sell tokens to the exchange at a relatively high price.

4. Preventing manipulation

The preceding analysis suggests that there is a possibility for platforms to avoid manipulation. They can do so by limiting the size of their buyback pledges. We can focus on the case where a single investor holds all of the tokens not held by users. In this scenario, there are no other investors who share in the benefits of the investor's actions to restrict the number of tokens available to the exchange. Hence, this is the situation where the incentives for price manipulation are the greatest.

Proposition 3, which considers the scenario of a single investor, shows that platforms that face lower user demand and promise to buy back a larger share of the issued tokens are the most vulnerable to manipulation. In the limit, where the user demand approaches zero, then the tokens are vulnerable if the fraction of share of coins that the exchange pledges to buy back exceeds one-half. As a simple rule-of-thumb, this suggests that, for any number of investors, in order to avoid manipulation an exchange should not pledge to buy back more than half of the tokens. Interestingly, Table 1 shows that 50% is the mode percentage for buy back pledges, and the majority of platform do adhere to this rule of thumb.

Limiting the share of tokens that the exchange pledges to buy back also limits the amount of funds it can raise by selling the tokens (Proposition 1). For an exchange that would like to attract more funding, it could be beneficial to explore whether it is possible to increase the user demand instead, for example by offering more benefits to tokens users. This would increase both the amount of funding it could raise and the threshold buyback amount at which the token would become vulnerable to manipulation by investors.

VII. Users with Investment Motives

In this section, we consider an extension where user demand for tokens is a function of the rate of appreciation of token prices, as in Prat et al. (2024). We do this for scenarios where the buyback program is full. Such scenarios are realistic (e.g., Bitfinex) and they allow for analytical solutions. Our result regarding the impact of buyback pledges on token prices survives this extension, although the amount an exchange can raise by selling tokens increases even if the exchange pledges relative few resources to the buyback program. Before, the buyback pledges had no impact on the amount raised unless the pledged resources would be sufficient to entice investors.

Users of crypto exchange tokens face a cash-in-advance constraint for tokens and a cost of capital for funds invested in tokens, r. They face sudden opportunities to use crypto exchange tokens such as paying for transaction fees. The opportunities occur randomly according to a Poisson process with intensity $\phi > 0$. The utility of the user with an opportunity to use crypto exchange tokens is a function of the dollar amount of crypto exchange tokens that is spent. The utility function $v(c)$ satisfies the usual conditions: It is continuously differentiable, $v_c(c) > 0$, $v_{cc}(c) < 0$, $v_c(c) \to 0$ as $c \to \infty$, and $v_c(c) \to \infty$ as $c \to 0$. Prat et al. (2024) derive that the following condition must hold true for the equilibrium dollar amount c^* of tokens that a user holds:

$$r = \phi\left(v_c(c^*) - 1\right) + \mu(t), \tag{2}$$

where $v_c(\cdot)$ denotes the marginal utility of spending a dollar amount of tokens when an opportunity occurs, and where $\mu(t)$ is the expected rate of appreciation $\mathbb{E}_t\left[\partial S(t)/\partial t\right]/S(t)$.

The intuition behind condition (2) is as follows. If the rate of appreciation equals the cost of capital ($\mu(t) = r$), then the marginal utility of spending a dollar of tokens equals the cost of purchasing a dollar of tokens. If the rate of appreciation is lower ($\mu(t) < r$), then the user faces an opportunity cost of holding tokens. The opportunity cost to the user equals the difference between the cost of capital and the expected rate of appreciation. The opportunity cost induces the user to hold less tokens.

Let n denote the initial number of users. Let $A(t)$ represent the growth in the number of users on the platform. Then the equilibrium transactional demand for tokens by all users for a given rate of appreciation can be calculated as

$$X^{\$}(t) = A(t)nc^{*}(t) = A(t)nv_c^{-1}\left(\frac{r - \mu(t) + \phi}{\phi}\right), \tag{3}$$

where the last equality holds true because of (2), and where $v_c^{-1}(\cdot)$ denotes the inverse of the marginal utility function, which is a decreasing function of its argument.

The equilibrium transactional demand for tokens has become a function of the rate of appreciation. A higher rate of appreciation is associated with a higher level of token balances per user, $v_c^{-1}(\cdot)$, and, hence, a higher transactional demand. The role of the rate of appreciation plays a smaller role if opportunities to spend tokens occur more frequently, that is, for higher values of ϕ. The opportunity cost to the users of holding tokens from one spending opportunity until the next is smaller when spending opportunities occur more frequently. Hence, the higher the likelihood of an opportunity to use tokens the smaller the impact of the opportunity cost on the user demand.

The equilibrium transactional demand in (3) takes the familiar form $X^{\$}(t) = A(t)X^{\$}$ for stationary levels of $\mu(t)$. By nesting the level of $X^{\$}$ obtained from (3) into the equilibrium path of the exchange rate in Proposition 1, we obtain the following lemma:

Lemma 1 *Suppose the transactional demand is endogenous and the buyback program is full. If* $Y^{\$} > (r + \lambda - g)nv_c^{-1}(1)$*, then* $X^{\$} = nv_c^{-1}(1)$ *and the initial equilibrium exchange rate equals*

$$S(0) = \frac{1}{M}\frac{Y^{\$}}{r + \lambda - g}.$$

Otherwise, the initial equilibrium exchange rate equals $S(0) = X^{\$}/M$*, with* $X^{\$}$ *implicitly defined by*

$$X^{\$} = nv_c^{-1}\left(\frac{r + \lambda - Y^{\$}/X^{\$} - g + \phi}{\phi}\right).$$

The lemma shows that the equation for the exchange rate is the same as before if the exchange pledges sufficient resources such that investors are willing to hold the tokens (see Corollary 1). The level of the exchange rate is again reminiscent of the Gordon growth model. Moreover, the level of transactional demand is as if there is no opportunity cost of holding tokens. If only users are willing to hold the tokens, then the results are different. In this situation, the exchange rate will depend on the way the buyback pledge affects the transactional demand. If users have investment motives, then pledging even a small amount of resources can impact the exchange rate, and, hence, the amount the exchange can raise by selling tokens.

The following proposition summarizes the impact of the amount of resources pledged by the exchange for buybacks for the utility function in general form.

Proposition 5 *Suppose the transactional demand is endogenous and the buyback program is full. Then the utility-demand* $X^{\$}$ *and the amount the exchange can raise by selling tokens,* $MS(0)$*, are strictly increasing in the amount of resources pledged for buybacks,* $Y^{\$}$*.*

The result can be illustrated with analytical solutions for the special case of log-utility.

Example 1 *Let the utility function of users take the form* $v(c) = log(c)$*. If* $Y^{\$} > (r+\lambda-g)n$*, then* $X^{\$} = n$ *and the initial equilibrium exchange rate equals*

$$S(0) = \frac{1}{M}\frac{Y^{\$}}{r + \lambda - g}.$$

Otherwise, the initial equilibrium exchange rate equals $S(0) = X^{\$}/M$*, with* $X^{\$}$ *defined by*

$$X^{\$} = \frac{n\phi + Y^{\$}}{r + \lambda - g + \phi}.$$

The initial exchange rate increases strictly in the amount of resources pledged for buybacks in line with Proposition 5. Moreover, for the case of log-utility we also find that the additional amount of funding that can be raised by including a buyback pledge is less than the discounted cost of the buyback. Starting from a point where no resources are pledged at all, one can use the expression for $X^{\$}$ in Example 1 to show that, if the discounted cost of resources pledged towards buybacks increases, then the initial utility-demand increases by

$$\frac{r + \lambda - g}{r + \lambda - g + \phi} < 1,$$

where the strict inequality holds since $\phi > 0$.[8] Finally, it is worth noting that, once the amount of resources pledged to the buyback reaches the level where investors are willing to hold the tokens, then any further increase in the amount of resources pledged to buybacks will increase the amount of funding that the exchange can raise by the same amount.

VIII. Concluding Remarks

We show that buyback pledges significantly impact price dynamics of crypto exchange tokens and can, under some circumstances, lead to dramatic price shifts. Buyback pledges are an inherently costly way for a platform owner to raise funds. Crucially, they can also be exploited by large investors, or coordinated groups of small investors, to the detriment of the platform owner. For both these reasons, buybacks are not the preferred form of funding for platforms and their existence should be seen as an indicator of capital market restrictions faced by platform developers or other frictions.

[8] Note that the discounted cost to the exchange of increasing $Y^{\$}$ by one dollar equals $1/(r + \lambda - g)$.

The possibility of rapid price movements from coordinated actions by investors fuelled through social media platforms was made evident by the GameStop "short squeeze" in January of 2021. This episode resulted in major short sellers covering their short positions after incurring significant losses, trading restrictions at broker-dealers, and intraday margin calls by clearing agencies (SEC Staff Report, 2021). In the case of crypto tokens, coordination among investors might be an even greater threat as it can be enforced through smart contracts.

Appendix: Proofs

A. Proof of Proposition 1

Proof.

First, we derive the following lemma.

Lemma 2 *At any time t, the exchange rate of the tokens equals*

$$S(t) = \frac{X^{\$}(t)}{M - W(0,t) - Z(t)}.$$

Proof. Rewriting the market clearing condition in Eq. (1) gives the equation. ■

Part 1: $(Y\$ > (r + \lambda - g)X^{\$})$

Consider the price of tokens if no collapse occurred before time T so that $A(t) > 0$ for any $t \leq T$. At time $t \geq T$, the exchange has bought back the tokens it promised to buy back, hence, $W(0,t) = U$ by definition, and there will be no incentives for investors to hold tokens, so $Z(t) = 0$. Using the expression in Lemma 2, the expected future equilibrium exchange rate in terms of dollars per coin conditional upon no collapse occurring can be written as

$$\mathbb{E}_t(S(T)|A(T) > 0) = \frac{X^{\$}e^{gT}}{M - U}.$$

Similarly, from Lemma 2, we must have that $\mathbb{E}_t(S(T)|A(T)=0)=0$ if a collapse occurred until time T such that $A(T)=0$.

Assume that the speculative position $Z(t)>0$ for any $t \leq T$ (we will verify under which condition this assumption holds true). For any time $t \leq T$ where $A(t)>0$, investors will be indifferent between holding and selling tokens if and only if

$$\begin{aligned}S(t) &= e^{-r(T-t)}\mathbb{E}_t S(T) = e^{-r(T-t)}\Pr(A(T)>0|A(t)>0)\mathbb{E}_t(S(T)|A(T)>0),\\ &= e^{-(r+\lambda)(T-t)}\mathbb{E}_t(S(T)|A(T)>0),\\ &= e^{-(r+\lambda)(T-t)}\frac{X^{\$}e^{gT}}{M-U}.\end{aligned} \tag{4}$$

Hence, provided that some investors are willing to hold the tokens (i.e., the assumption $Z(t)>0$ for $0 \leq t < T$) and $A(t)>0$, the exchange rate at $t \leq T$ is given by this expression.

What remains to be solved is the moment when the exchange has finished buying back the tokens, i.e., the value of T. The value the exchange spends on tokens between t_1 and t_2 equals $\int_{t_1}^{t_2} Y^{\$}(t)dt$ for any $t_1, t_2 \leq T$. Given the expression for $S(t)$ in (4), the total number of tokens the exchange buys back between t_1 and t_2 equals

$$\begin{aligned}W(t_1,t_2) &= \int_{t_1}^{t_2}\frac{Y^{\$}(t)}{S(t)}dt = \int_{t_1}^{t_2}(M-U)\frac{Y^{\$}e^{gt}}{X^{\$}e^{gT}}e^{+(r+\lambda)(T-t)}dt\\ &= \int_{t_1}^{t_2}(M-U)\frac{Y^{\$}}{X^{\$}}e^{+(r+\lambda-g)(T-t)}dt\\ &= (M-U)\frac{Y^{\$}}{X^{\$}}e^{(r+\lambda-g)T}\left(\frac{e^{-(r+\lambda-g)t_1}-e^{-(r+\lambda-g)t_2}}{r}\right).\end{aligned}$$

Using the definition of T, i.e., the time at which the exchange has bought back all tokens it promised to buy back, we know $W(0,T)=U$. Using this in the above expression with $(t_1,t_2)=(0,T)$ gives

$$U = (M-U)\frac{Y^{\$}}{(r+\lambda-g)X^{\$}}\left(e^{(r+\lambda-g)T}-1\right),$$

which has the unique solution for T given in the proposition.

For the proposition to be an equilibrium outcome, we still need to verify under which condition the assumption of a positive speculative position, i.e., $Z(t) > 0$ for any $t < T$, holds true. In order to do so, we first obtain the expression for $S(T)$ in the following lemma by plugging the solution for T into Eq. (4).

Lemma 3 *Suppose no collapse occurs before time t. The equilibrium exchange rate at $0 \leq t < T$ in the price equilibrium where $Z(t) > 0$ for $0 \leq t < T$ equals*

$$S(t) = e^{(r+\lambda)t} \frac{X^{\$} Y^{\$}}{(M-U)Y^{\$} + U(r+\lambda-g)X^{\$}}.$$

Moreover, given the solution for T, we have

$$W(0,t) = (1 - e^{-(r+\lambda-g)t}) \left(U + \frac{Y^{\$}}{(r+\lambda-g)X^{\$}}(M-U) \right).$$

We can back out the value of $Z(t)$ from plugging this expression for $W(0,t)$ into the exchange rate equation in Lemma 2 and it equal to the solution for $S(t)$ in Lemma 3, i.e.,

$$e^{(r+g)t} \frac{X^{\$} Y^{\$}}{(M-U)Y^{\$} + U(r+\lambda-g)X^{\$}} = \frac{X^{\$}}{M - (1 - e^{-(r+\lambda-g)t}) \left(U + \frac{Y^{\$}}{(r+\lambda-g)X^{\$}}(M-U) \right) - Z(t)}.$$

Solving this equation for $Z(t)$ gives

$$\begin{aligned} Z(t) =& e^{-(r+\lambda-g)t} \left[U \frac{Y^{\$} - (r+\lambda-g)X^{\$}}{Y^{\$}} + (M-U) \frac{Y^{\$} - (r+\lambda-g)X^{\$}}{(r+\lambda-g)X^{\$}} \right] \\ & - \left[\frac{Y^{\$} - (r+\lambda-g)X^{\$}}{(r+\lambda-g)X^{\$}} (M-U) \right]. \end{aligned} \tag{5}$$

This gives $Z(t) > 0$ if

$$e^{-(r+\lambda-g)t}\left[U\frac{Y^{\$}-(r+\lambda-g)X^{\$}}{Y^{\$}}+(M-U)\frac{Y^{\$}-(r+\lambda-g)X^{\$}}{(r+\lambda-g)X^{\$}}\right] > \left[\frac{Y^{\$}-(r+\lambda-g)X^{\$}}{(r+\lambda-g)X^{\$}}(M-U)\right],$$

or, conditional upon $Y^{\$} > (r+\lambda-g)X^{\$}$,

$$e^{-(r+\lambda-g)t}\left[1+\frac{U}{M-U}\frac{(r+\lambda-g)X^{\$}}{Y^{\$}}\right] > 1. \tag{6}$$

Solving this inequality for t gives that, conditional upon $Y^{\$} > (r+\lambda-g)X^{\$}$, this condition holds true for any $t < T$. So, we must have $Z(t) > 0$ for any $t < T$ if $Y^{\$} > (r+\lambda-g)X^{\$}$, which is the condition in the proposition. To prove that this condition is also necessary, note that $Z(t) = 0$ in Eq. (5) if we were to have $Y^{\$} = (r+\lambda-g)X^{\$}$ Moreover, using the same line of proof, we would find $Z(t) < 0$ for any $t < T$ if we were to have $Y^{\$} < (r+\lambda-g)X^{\$}$ as the inequality in (6) would be reversed. Hence, $Y^{\$} > (r+\lambda-g)X^{\$}$ is a sufficient and a necessary condition.

Part 2: ($Y\$ \leq (r+\lambda-g)X^{\$}$)

Assume that $Z(t) = 0$ for any time t (we will verify under which conditions this assumption holds true). From $Z(t) = 0$ for $0 \leq t < T$ and the expression in Lemma 2, we have

$$S(t) = \frac{X^{\$}e^{gt}}{M-W(0,t)}. \tag{7}$$

The number of tokens that are bought back by the exchange between t_1 and t_2 for $t_1, t_2 \leq T$ equals

$$W(t_1,t_2) = \int_{t_1}^{t_2}\frac{Y^{\$}e^{gt}}{S(t)}dt = \int_{t_1}^{t_2}\frac{Y^{\$}}{X^{\$}}(M-W(0,t))dt.$$

Setting $(t_1, t_2) = (0, t)$ and taking a derivative on both sides with respect to t gives the ordinary differential equation

$$\frac{dW(0,t)}{dt} = \frac{Y^\$}{X^\$}(M - W(0,t)).$$

Reordering and integrating gives

$$\int \frac{1}{M - W(0,t)} dW(0,t) = \int \frac{Y^\$}{X^\$} dt,$$

which solves to

$$-\log\left(M - W(0,t)\right) + \log(c_0) = \frac{Y^\$}{X^\$} t,$$

for some arbitrary constant c_0. Using the fact that $W(0,0) = 0$ and solving for c_0 gives $c_0 = M$. Rewriting and ordering gives the solution for $W(0,t)$ as

$$W(0,t) = M - Me^{-tY^\$/X^\$}.$$

Using the expression for $W(0,t)$ to solve $W(0,T) = U$ for T gives the value of T in the proposition. The formula for the exchange rate in the proposition follows from plugging the solution for $W(0,t)$ into Eq. (7).

What remains to be proven is under which conditions the assumption $Z(t) = 0$ holds true. Conditional upon no collapse, the exchange rate of tokens in the proposition appreciates at a rate $g + Y^\$/X^\$$ for $0 \leq t < T$. Hence, the unconditional expected rate of appreciation is $g + Y^\$/X^\$ - \lambda$ (similar derivation as in part 1 of the proof of Proposition 1). Investors would not be willing to invest in tokens if the expected rate of appreciation would be less than the cost of capital, that is, if $g + Y^\$/X^\$ - \lambda < r$. Hence, the equilibrium described in Proposition 1 can be an equilibrium for $Z(t) = 0$ if $(r + \lambda - g)X^\$ > Y^\$$. Conversely, the equilibrium described in Proposition 1 cannot be an equilibrium for $Z(t) = 0$ if $(r + \lambda - g)X^\$ < Y^\$$ because investors would be willing to hold the tokens as the expected rate of appreciation,

$g+Y^\$/X^\$-\lambda$, would be higher than the cost of capital r. Finally, the proof of Proposition 1 showed that $Z(t)=0$ in the case where $Y^\$=(r+\lambda-g)X^\$$. Hence, a necessary and sufficient condition for the assumption for $Z(t)=0$ to hold true in equilibrium is $Y^\$\leq(r+\lambda-g)X^\$$ which is the condition in the proposition. ■

B. *Proof of Proposition 2*

Proof.

Part 1: $(Y\$ > (r+\lambda-g)X^\$)$

The cost of the buyback program in terms of the present value of the buybacks if $Y^\$ > (r+\lambda-g)X^\$$ equals

$$C(t_1,t_2)=\int_{t_1}^{t_2} e^{-(r+\lambda-g)t}Y^\$dt=\frac{e^{-(r+\lambda-g)t_1}-e^{-(r+\lambda-g)t_2}}{(r+\lambda-g)}Y^\$. \tag{8}$$

Calculating this value for $(t_1,t_2)=(0,T)$ where T equals the solution for T in Proposition 1 gives the expression for the present value of the buybacks as

$$C=U\frac{X^\$Y^\$}{(M-U)Y^\$+U(r+\lambda-g)X^\$}=US(0).$$

The incremental amount raised with the buyback pledge minus the expected costs equals

$$\begin{aligned}MS(0)-X^\$-C&=(M-U)S(0)-X^\$,\\&=\frac{(M-U)X^\$Y^\$-\left[(M-U)Y^\$+U(r+\lambda-g)X^\$\right]X^\$}{(M-U)Y^\$+U(r+\lambda-g)X^\$},\\&=-\frac{U(r+\lambda-g)X^\$X^\$}{(M-U)Y^\$+U(r+\lambda-g)X^\$}.\end{aligned}$$

The last expression is strictly less than zero for any $U>0$.

Part 2: $(Y\$ < (r+\lambda-g)X^\$)$

If $Y^\$ \leq (r+\lambda-g)X^\$$, then the expected cost of the buyback program can be obtained by plugging the solution for the value of T in Proposition 1 for the utility-regime into Eq. (8) for the expected buyback costs gives

$$C(0,T) = \frac{e^{-(r+\lambda-g)\times 0} - e^{-(r+\lambda-g)\frac{X^\$}{Y^\$}\log\left(\frac{M-U}{M}\right)}}{r+\lambda-g}Y^\$,$$

which, after rewriting, gives

$$C = \frac{Y^\$}{r+\lambda-g} - \frac{Y^\$}{r+\lambda-g}\left(\frac{M-U}{M}\right)^{\frac{(r+\lambda-g)X^\$}{Y^\$}}.$$

The incremental amount raised with the buyback pledge minus the expected costs equals

$$MS(0) - X^\$ - C = X^\$ - X^\$ - C = -C,$$

which is less than zero since $C > 0$ for $U > 0$. ■

C. *Proof of Proposition 3*

Proof. If no strategic actions are anticipated by others, then the exchange rate at $t = 0$, before the tokens are permanently frozen, follows immediately from Lemma 3. The holdings by users at $t = 0$ equal

$$\frac{X^\$}{S(0)} = M - U\frac{Y^\$ - (r+\lambda-g)X^\$}{Y^\$},$$

and, since $W(0,0) = 0$, we can calculate the initial position of the investor at $t = 0$ by subtracting the holdings of users from M as

$$Z(0) = U\frac{Y^\$ - (r+\lambda-g)X^\$}{Y^\$}. \tag{9}$$

The strategy is feasible if and only if the initial position of the Comparing $M - U$ to the expression for $Z(0)$, the strategy is feasible without price impact before freezing the tokens if and only if

$$(r + \lambda - g)X^{\$} < 2Y^{\$}\left(1 - \frac{M}{2U}\right),$$

which is equivalent to the first condition in the proposition.

After permanently freezing $M - U$ tokens, a new equilibrium will be reached at exchange rate $S'(0)$. The new equilibrium situation is equivalent to that where the exchange intends to buy back all existing described in Corollary 1 except that the number of tokens equals $M - (M - U) = U$. Using U instead of M in the equation for the exchange rate in Corollary 1 gives the expression for the exchange rate in the new equilibrium as

$$S'(0) = \frac{1}{U}\frac{Y^{\$}}{r + \lambda - g}.$$

It is profitable for the investor to permanently freeze $M - U$ coins if and only if the value of the remaining coins under the new equilibrium price exceed the total value of all coins at the exchange rate without freezing the coins, i.e., if and only if

$$\left[Z(0) - (M - U)\right] S'(0) > Z(0)S(0).$$

For the profitability of the strategy, it does not matter whether the investor retains the remaining coins or whether she sells them to other investors after reaching the new equilibrium. Using the expression for $Z(0)$ in (9), and the expressions for $S(0)$ and $S'(0)$, gives

$$\left[U\frac{Y^{\$} - (r + \lambda - g)X^{\$}}{Y^{\$}} - (M - U)\right]\left[\frac{Y^{\$}}{U(r + \lambda - g)}\right] > \left[U\frac{Y^{\$} - (r + \lambda - g)X^{\$}}{Y^{\$}}\right]\left[\frac{X^{\$}Y^{\$}}{(M - U)Y^{\$} + U(r + \lambda - g)X^{\$}}\right],$$

or,

$$\frac{(2U-M)Y^{\$}-U(r+\lambda-g)X^{\$}}{U(r+\lambda-g)} > \frac{X^{\$}U(Y^{\$}-(r+\lambda-g)X^{\$})}{(M-U)Y^{\$}+U(r+\lambda-g)X^{\$}}.$$

Rewriting gives

$$\left[(U-M)Y^{\$}+U(Y-(r+\lambda-g)X^{\$})\right]\left[(M-U)Y^{\$}+U(r+\lambda-g)X^{\$}\right] >$$
$$\left[U(Y^{\$}-(r+\lambda-g)X^{\$})\right]\left[U(r+\lambda-g)X^{\$}\right],$$

or,

$$\left[U(Y^{\$}-(r+\lambda-g)X^{\$})\right]\left[(M-U)Y^{\$}\right]-\left[(M-U)Y^{\$}\right]\left[U(r+\lambda-g)X^{\$}\right] > \left[(M-U)Y^{\$}\right]^2.$$

Dividing both sides by $(M-U)Y^{\$}$ and rewriting gives the condition as

$$U(2Y^{\$}-2(r+\lambda-g)X^{\$}) > MY^{\$},$$

which is equivalent to the second condition in the proposition. ■

D. Proof of Proposition 4

Proof. The strategy is profitable to the largest investor if

$$\left[\omega_1 Z(0)-(M-U)\right]S'(0) > \omega_1 Z(0)S(0).$$

Using the expression for $Z(0)$ in (9), and the expressions for $S(0)$ and $S'(0)$, gives

$$\left[\omega_1 U\frac{Y^{\$}-(r+\lambda-g)X^{\$}}{Y^{\$}}-(M-U)\right]\left[\frac{Y^{\$}}{U(r+\lambda-g)}\right] >$$
$$\left[\omega_1 U\frac{Y^{\$}-(r+\lambda-g)X^{\$}}{Y^{\$}}\right]\left[\frac{X^{\$}Y^{\$}}{(M-U)Y^{\$}+U(r+\lambda-g)X^{\$}}\right],$$

or,

$$\left[\omega_1 U(Y - (r+\lambda-g)X^{\$}) - (M-U)Y^{\$}\right]\left[(M-U)Y^{\$} + U(r+\lambda-g)X^{\$}\right] >$$
$$\left[\omega_1 U(Y^{\$} - (r+\lambda-g)X^{\$})\right]\left[U(r+\lambda-g)X^{\$}\right],$$

which can be rewritten to,

$$\left[\omega_1 U(Y^{\$} - (r+\lambda-g)X^{\$})\right]\left[(M-U)Y^{\$}\right] > \left[(M-U)Y^{\$}\right]^2 + \left[(M-U)Y^{\$}\right]\left[U(r+\lambda-g)X^{\$}\right].$$

Dividing both sides by $(M-U)Y^{\$}$ and rewriting gives $\omega_1 > \omega^*$, where ω^* is defined in the proposition. ■

E. Proof of Proposition 5

Proof. For $Y^{\$} > (r+\lambda-g)nv_c^{-1}(1)$, it is immediate from Lemma 1 that $dX^{\$}/dY^{\$} > 0$. What remains to be proven is that $dX^{\$}/dY^{\$} > 0$ if $Y^{\$} \leq (r+\lambda-g)nv_c^{-1}(1)$. Differentiating both sides of the utility-demand in Lemma 1 for this scenario with respect to $Y^{\$}$ gives

$$\frac{dX^{\$}}{dY^{\$}} = n \cdot (v_c^{-1})' \left(\frac{r+\lambda-Y^{\$}/X^{\$}-g+\phi}{\phi}\right) \cdot \frac{\frac{dX^{\$}}{dY^{\$}}Y^{\$} - X^{\$}}{(X^{\$})^2\phi},$$

where $(v_c^{-1})'$ denotes the derivative of the inverse of the marginal utility function. Bringing $dX^{\$}/dY^{\$}$ to the left-hand-side gives

$$\frac{dX^{\$}}{dY^{\$}}\left(1 - n\cdot(v_c^{-1})'\left(\frac{r+\lambda-Y^{\$}/X^{\$}-g+\phi}{\phi}\right)\cdot\frac{Y^{\$}}{X^{\$}}\frac{1}{\phi X^{\$}}\right) = -n\cdot(v_c^{-1})'\left(\frac{r+\lambda-Y^{\$}/X^{\$}-g+\phi}{\phi}\right)\cdot\frac{1}{\phi X^{\$}},$$

which solves to the following differential equation

$$\frac{dX^{\$}}{dY^{\$}} = \frac{-(v_c^{-1})'\left(\frac{r+\lambda-Y^{\$}/X^{\$}-g+\phi}{\phi}\right)}{\phi\frac{X^{\$}}{n} - (v_c^{-1})'\left(\frac{r+\lambda-Y^{\$}/X^{\$}-g+\phi}{\phi}\right)\frac{Y^{\$}}{X^{\$}}}. \tag{10}$$

Note that $(v_c^{-1})'$ is strictly negative since the inverse function theorem implies $(v_c^{-1})' = 1/v_{cc}$ and $v_{cc} < 0$. Hence, the numerator of (10) must be strictly positive. Similarly, the denominator of (10) must be strictly positive since $\phi, X^{\$}, n, Y^{\$} > 0$. This proves $dX^{\$}/dY^{\$} > 0$ and hence $S(0)$ is increasing in $Y^{\$}$ if $Y^{\$} \leq (r + \lambda - g)nv_c^{-1}(1)$. ■

F. Proof of Example 1

Proof. If $v(c) = log(c)$, then $v_c(c) = 1/c$ and $v_c^{-1} = 1/v_c$. Then, from the equation for the equilibrium transactional demand in (3), we obtain

$$X^{\$} = \frac{n\phi}{\phi + r - \mu(t)}.$$

If $Y^{\$} > (r + \lambda - g)n$, then $\mu(t) = r$, and $X^{\$} = n$ since $v_c^{-1}(1) = 1$. Otherwise, $\mu(t) = g + Y^{\$}/X^{\$} - \lambda$, and we have

$$X^{\$} = \frac{n\phi}{r + \lambda - Y^{\$}/X^{\$} - g + \phi},$$

which solves for a unique value of $X^{\$}$,

$$X^{\$}(r + \lambda - g + \phi) - Y^{\$} = n\phi,$$

and hence

$$X^{\$} = \frac{n\phi + Y^{\$}}{r + \lambda - g + \phi}.$$

To verify whether this is an equilibrium we need to verify whether the condition $Y^{\$} \leq (r + \lambda - g)nv_c^{-1}(1)$ holds true, where $v_c^{-1}(1) = 1$. Rewriting the expression for $X^{\$}$ gives $Y^{\$} = (r + \lambda - g + \phi)X^{\$} - n\phi$. Note that $X^{\$} < n$ since the maximum possible value of $X^{\$}$ if the rate of appreciation equals r is n. This confirms $Y^{\$} \leq (r + \lambda - g)nv_c^{-1}(1)$. ■

References

A. Aloosh and J. Li. Direct Evidence of Bitcoin Wash Trading. *Management Science*, forthcoming.

J. Aoyagi and Y. Ito. Coexisting Exchange Platforms: Limit Order Books and Automated Market Makers. *Working Paper*, 2021.

S. Athey, I. Parashkevov, V. Sarukkai, and J. Xia. Bitcoin Pricing, Adoption, and Usage: Theory and Evidence. *Working Paper*, 2016.

Y. Bakos and H. Hałaburda. The Role of Cryptographic Tokens and ICOs in Fostering Platform Adoption. *Information Systems Research*, 33(2):1368–1385, 2022.

B. Biais, C. Bisière, M. Bouvard, C. Casamatta, and A. Menkveld. Equilibrium Bitcoin Pricing. *Journal of Finance*, 78(2):967–1014, 2023.

Binance. Binance Exchange. *White Paper Version 1.1*, 2018. URL https://tinyurl.com/bd9cjw2x.

Bitfinex. Initial Exchange Offering of LEO Tokens. *White Paper (May 8)*, 2019a. URL https://www.bitfinex.com/wp-2019-05.pdf.

Bitfinex. UNUS SED LEO to be Listed on Bitfinex on Monday Following Completion of $1Bn Contribution. *Announcement on Medium.com (May 17)*, 2019b. URL http://tiny.cc/st7bvz.

W. Bolt and M.R.C. Van Oordt. On the Value of Virtual Currencies. *Journal of Money, Credit and Banking*, 52(2):835–862, 2020.

J. Chiu and T.V. Koeppl. The Economics of Cryptocurrencies: Bitcoin and Beyond. *Canadian Journal of Economics*, 55(4):1762–1798, 2022.

J. Chiu and T.-N. Wong. Payments on Digital Platforms: Resiliency, Interoperability and Welfare. *Journal of Economic Dynamics and Control*, 142:104,173, 2022.

CoinGecko. Where are Crypto Exchanges Incorporated? *Analysis (October 23)*, 2023.

L.W. Cong, Y. Li, and N. Wang. Tokenomics: Dynamic Adoption and Valuation. *Review of Financial Studies*, 34(3):1105–1155, 2021.

L.W. Cong, Y. Li, and N. Wang. Token-Based Platform Finance. *Journal of Financial Economics*, 144(3):972–991, 2022.

L.W. Cong, X. Li, K. Tang, and Y. Yang. Crypto Wash Trading. *Management Science*, 69 (11):6427–6454, 2023.

Financial Times. New Binance Chief Refuses to Disclose Global Headquarters' Location. *News Report (December 5)*, 2023.

FTX. FTX. *White Paper*, 2020. URL https://tinyurl.com/48c5hu56.

R.J. Garratt and M.R.C. Van Oordt. Entrepreneurial Incentives and the Role of Initial Coin Offerings. *Journal of Economic Dynamics and Control*, 142:104171, 2022.

R.J. Garratt and M.R.C. Van Oordt. The Crypto Multiplier. *Bank for International Settlements Working Paper*, 1104, 2023.

R.J. Garratt and N. Wallace. Bitcoin 1, Bitcoin 2, ... : An Experiment on Privately Issued Outside Monies. *Economic Inquiry*, 56(3):1887–1897, 2018.

M.J. Gordon. Dividends, Earnings, and Stock Prices. *Review of Economics and Statistics*, 41(2):99–105, 1959.

S. Gryglewicz, S. Mayer, and E. Morellec. Optimal Financing with Tokens. *Journal of Financial Economics*, 142(3):1038–1067, 2021.

S. Karau and E. Moench. Is It Really Mine? Mining Shocks and their Effects on Bitcoin Valuations. *Working Paper*, 2023.

J. Lee and C.A. Parlour. Consumers as Financiers: Consumer Surplus, Crowdfunding, and Initial Coin Offerings. *Review of Financial Studies*, 35(3):1105–1140, 2022.

A. Lehar and C. Parlour. Decentralized Exchange: The Uniswap Automated Market Maker. *Journal of Finance*, forthcoming.

K. Malinova and A. Park. Tokenomics: When Tokens Beat Equity. *Management Science*, 69(11):6568–6583, 2023a.

K. Malinova and A. Park. Learning from DeFi: Would Automated Market Makers Improve Equity Trading? *Working Paper*, 2023b.

New York AG. Attorney General James Announces Court Order Against “Crypto” Currency Company Under Investigation For Fraud. *Press release (25 April)*, 2019. URL http://tiny.cc/gt7bvz.

E.S. Pagnotta. Decentralizing Money: Bitcoin Prices and Blockchain Security. *Review of Financial Studies*, 35(2):866–907, 2022.

J. Prat, V. Danos, and S. Marcassa. Fundamental Pricing of Utility Tokens. *Working Paper*, 2024.

K. Rogoff and Y. You. Redeemable Platform Currencies. *Review of Economic Studies*, 90 (2):975–1008, 2023.

L. Schilling and H. Uhlig. Some Simple Bitcoin Economics. *Journal of Monetary Economics*, 106:16–26, 2019.

SEC Staff Report. Staff Report on Equity and Options Market Structure Conditions in Early 2021. *Date: October 14*, 2021. URL https://tinyurl.com/46tz8wty.

M. Sockin and W. Xiong. Decentralization through Tokenization. *Journal of Finance*, 78 (1):247–299, 2023.

M.R.C. Van Oordt. On Bubbles in Cryptocurrency Prices. *Working Paper*, 2024.

Previous volumes in this series

1200 July 2024	Financial inclusion transitions in Peru: does labor informality play a role?	Jose Aurazo and Farid Gasmi
1199 July 2024	New spare tires: local currency credit as a global shock absorber	Stefan Avdjiev, John Burger and Bryan Hardy
1198 July 2024	Sovereign green bonds: a catalyst for sustainable debt market development?	Gong Cheng, Torsten Ehlers, Frank Packer and Yanzhe Xiao
1197 July 2024	The gen AI gender gap	Iñaki Aldasoro, Olivier Armantier, Sebastian Doerr, Leonardo Gambacorta and Tommaso Oliviero
1196 July 2024	Digital payments, informality and economic growth	Ana Aguilar, Jon Frost, Rafael Guerra, Steven Kamin and Alexandre Tombini
1195 July 2024	The asymmetric and persistent effects of Fed policy on global bond yields	Tobias Adrian, Gaston Gelos, Nora Lamersdorf, Emanuel Moench
1194 June 2024	Intelligent financial system: how AI is transforming finance	Iñaki Aldasoro, Leonardo Gambacorta, Anton Korinek, Vatsala Shreeti and Merlin Stein
1193 June 2024	Aging gracefully: steering the banking sector through demographic shifts	Christian Schmieder and Patrick A Imam
1192 June 2024	Sectoral heterogeneity in the wage-price pass-through: Evidence from Euro area	Miguel Ampudia, Marco Lombardi and Théodore Renault
1191 May 2024	The impact of macroprudential policies on industrial growth	Carlos Madeira
1190 May 2024	CEO turnover risk and firm environmental performance	Giulio Cornelli, Magdalena Erdem and Egon Zakrajsek
1189 May 2024	Sixty years of global inflation: a post GFC update	Raphael Auer, Mathieu Pedemonte and Raphael Schoenle
1188 May 2024	Finding a needle in a haystack: a machine learning framework for anomaly detection in payment systems	Ajit Desai, Anneke Kosse and Jacob Sharples
1187 May 2024	Nothing to hide? Gender and age differences in willingness to share data	Olivier Armantier, Sebastian Doerr, Jon Frost, Andreas Fuster and Kelly Shue

All volumes are available on our website www.bis.org.